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DISCUSSION OF HELIOMETER-OBSERVATIONS

OF

JUPITER'S SATELLITES,

MADE BY SIR DAVID GILL K. C. B. AND W. H. FINLAY M. A.

BY

W. DE SITTER.

GRONINGEN, J. B. WOLTERS, 1901.

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INTRODUCTION.

From more than one point of view the theory of the system of Jupiter is one of the most interesting problems in celestial mechanics, but, strange to say, it has also been one of the most neglected subjects during the last half century. During the eighteenth century and the first half of the nineteenth the attention of astronomers had been largely directed towards the motion of the satellites. The eclipses had been diligently observed from the time that telescopes existed powerful enough to show these phenomena, and many of the periodic inequalities had been discovered and represented by empirical formulae.

LAPLACE, in the fourth volume of the *Mécanique Céleste*, developed the mathematical theory of the mutual perturbations of the satellites. His formulae were reduced to numbers by BOUVARD, but in this form they still contained 31 unknown quantities, *viz.* the elements and masses of the four satellites, the compression of Jupiter, and the inclination and node of its equator. The immense work of deriving the values of these unknowns from the observations was first undertaken by DELAMBRE, who in 1792 published tables based on the discussion of over 2000 eclipses. Afterwards he applied slight corrections to his coefficients. The corrected tables were based on all the eclipses (about 6000) observed previous to 1804. These tables appear to have represented the observations with sufficient accuracy at the time of their publication.

From Delambre's coefficients Laplace derived the elements and masses of the satellites and the value of the coefficient J of the compression of Jupiter.

After Delambre, DAMOISEAU took up the subject, and in 1836 published his *Tables écliptiques des satellites de Jupiter*, on which, up to the present date, nearly all computations referring to the satellites (especially the prediction of the eclipses and other phenomena in the national ephemerides) have been based. [1])

The tables are however by no means so perfect as might be inferred from this general adoption; it is well known that large discrepancies between the predicted and observed time of an eclipse are not at all uncommon.

In the first place, it is not known upon what observations they are based, or how Damoiseau derived the values of the coefficients of the inequalities. But the most serious defect of the tables, which makes that they can hardly be used with confidence as a means of comparing the observations with theory, is the fact, that they do not represent a consistent theory. Some of the important inequalities have values which differ very little from those of Delambre's tables, while others are considerably changed, and it is to be doubted, whether the different inequalities can be represented by one same set of elements and masses.

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1) DAMOISEAU's tables only went to 1880. They were continued by TODD to 1900 (published at *Washington* 1876), and by ADAMS (*Appendix to the* N. A. *for 1881 and 1900*) to 1910.

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Damoiseau, in his introduction, gives values for the masses, but these are inconsistent with the coefficients of the tables. Adams[1]) substituted the coefficients from the tables in the equations by which Laplace derived the masses from Delambre's coefficients, and the resulting masses do not agree with those given by Damoiseau. This will be seen from the following comparison of the masses adopted by different investigators.

MASSES OF SATELLITES AND COEFFICIENT J.

	Laplace	Damoiseau	Adams
m_1	0.0000 173 281	168 770	283 113
m_2	0.0000 232 355	232 270	232 355
m_3	0.0000 884 972	884 370	812 453
m_4	0.0000 426 591	424 751	214 880
J	0.0 219 013	247 187	216 623

The urgent need of better tables has often been felt, and was expressed by Bessel in 1842, *only six years after the publication of Damoiseau's tables.* He writes (*Astronomische Untersuchungen*, II, page 25): „Ich fürchte, dass man, um die Formeln für die Bewegungen der Jupiterssatelliten wieder theoretisch *ganz zusammenhangend* zu machen, gezwungen sein wird, die unvollständig dargestellten Resultate des Herrn *Damoiseau* zu verlassen und bis zu einer neuen Untersuchung der Elemente zurückzugehen."

In 1880 Souillart published his beautiful *Théorie analytique des Satellites de Jupiter* (*Memoirs* R. A. S., *Vol.* XLV), which is in many points more complete than Laplace's theory. That Souillart when he made his theory had the same idea, which is expressed as above by Bessel, cannot be better shewn than in his own words (*l. c. page* 1): "Je présente ici, coordonnées dans une rédaction d'ensemble, toutes mes recherches sur cette question, dans le but d'offrir aux astronomes une théorie analytique complète des mouvements de ces satellites, pouvant servir de base à une nouvelle détermination des éléments et à la construction de nouvelles tables."

When we come to examine what observations are available as the basis for such a new determination of the elements, we find in the first place an immense number of observations of eclipses. But it is evident almost at first sight that we want something more than that. Among the worst determined of the elements are the inclinations and nodes of the orbits. These can be derived from eclipse observations only by the discussion of the duration of the eclipses. But this duration also depends on other unknowns, the influence of which cannot be separated from that of the positions of the orbits without a very considerable loss of weight. What is wanted are observations of the (jovicentric) *latitude* of the satellites in all points of their orbits.

Bessel (1833—39) observed with the Königsberg heliometer distances of all the satellites and position-angles of the third and fourth satellites from the planet. Airy (1832—36) determined differences of right-ascension and declination between the planet and satellite IV. Vogel (1868—70) observed differences of R. A. of all the satellites and the planet. Schur (1874—80) used the heliometer to measure distances and position-angles from the planet to the four satellites. All these observations however were undertaken with a view primarily to determine the mass of Jupiter. They are too few in number and too much affected by instrumental or personal errors, to serve as a sufficient basis for the construction of new tables of the satellites.

In 1891 Dr. (now Sir David) Gill made the series of heliometer observations of the satellites,

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1) Scientific Papers of John Couch Adams. Vol. II, page 191.

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which is discussed in the present work. He measured distances and position-angles *from one satellite to another*, observing the whole set of six combinations every night, as far as possible. Everybody who has ever tried to bring the images of a star and a planet's limb to coincidence in the field of a heliometer, will at once admit, that this plan is much better than Bessel's and Schur's method of measuring the distances of the satellite to the two opposite edges of the planet's disc. It is evident that the observation of two similar objects, such as two satellites, is in the first place much easier, and therefore affected with smaller *accidental* errors, than the observation of such dissimilar objects as the star-like image of a satellite and the more or less undulating limb of the planet. Not only so, but the observation of two satellites is much less liable to *systematic* errors, which may be, and probably are, different in different atmospheric definition, and which will not with certainty be eliminated in the mean of the measures to the two edges of the planet's disc.

These observations had remained unreduced till 1898. In the beginning of that year I was charged, in my capacity as a computer at the Royal Observatory at the Cape of Good Hope, with their reduction, and at the same time Dr. GILL encouraged me to take up the further discussion, and suggested this investigation as a suitable subject for my dissertation for the degree of D. Sc.

I need not endeavour here to express at length my gratitude to him for placing this rich material, probably the best observations he ever made, in my hands. It is only one instance of many kindnesses to me during all the time I stayed at the Cape. If my work, based on his observations, and inspired by his example and enthousiasm, is only in the remotest degree worthy of the kind and almost fatherly interest he always showed in it and in me, I shall consider the labour spent upon it amply repaid.

If ever the intention had existed to keep my own part of the work and that of the Cape Observatory distinctly separated, so that a sharp line could be drawn between one part and the other, this very soon became impossible by the kindness of Dr. GILL, who offered me the aid of members of the staff of the Cape Observatory for the examination of my computations and for the immense work of the formation and solution of the normal equations, thus relieving me of the greater part of the purely numerical work. I seize this opportunity to express my thanks to the several gentlemen who have taken part in this work. The share done by each of them will be aknowledged in the course of the discussion. I am also under a debt of gratitude to Prof. KAPTEYN, who not only on several occasions placed the forces of the staff of the Astronomical Laboratory at my disposal, but also allowed me to devote the greater part of my time during office-hours to this work. In addition to the different checks by sums or differences, computations in duplicate and examinations done by the computers at the Cape and Groningen, I have myself subjected all the computations to a thorough final revision.

In the present work I only intend to give the *results* of the discussion of the observations, and the explanation of the methods by which these results were derived, only adding in the first chapter so much about the methods of observing and about instrumental and personal errors as is necessary for the good understanding of the following chapters. The full details of the observations themselves, as well as of the greater part of the numerical work connected with the reduction and discussion, will soon be published as *Part* I *of Volume* VIII *of the Annals of the Royal Observatory at the Cape of Good Hope.*

From the statements already made it will be clear that the observations should not be compared with Damoiseau's tables. There is still another circumstance which would make it impossible to use Damoiseau's tables for the computation of tabular places, even if they were entirely trustworthy. Damoiseau has in his tables combined into one single term all those inequalities which at the epochs of the eclipses have the same argument, but different arguments at all other epochs. The observations must therefore be compared directly with Souillart's theory. Souillart has adopted the masses and elements given by Damoiseau, and with these fundamental constants has reduced his formulae to numbers. This second part

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of his work was published in 1887 (*Mémoires présentés par divers savants à l'académie des sciences de l'Institut de France, tome XXX*). The inequalities, which he derives do not agree of course with Damoiseau's tables.

From the comparison of the observed distances and position-angles with those computed from Souillart's theory it is possible to derive corrections to his adopted fundamental constants. It will be shewn however in the course of the discussion, that it is not possible to determine *all* the elements from these observations alone, but that the results here derived must be combined with those from other observations, taken at another epoch, in order to arrive at a complete determination of the fundamental constants of the theory. In the last chapter I will consider the question which kind of observations, and at what epoch, afford the most favourable conditions, in combination with the present series of observations, for such a determination of the elements, which would be sufficiently accurate to be made the basis of the formation of new tables

I cannot close this introduction without a few words to express my deep sense of gratitude to Prof. J. C. Kapteyn, who first by his kindling enthousiasm drew me towards the study of astronomy, who was my first teacher, and who now is the guide of my first step in my astronomical career. It will be largely due to him, and to Sir David Gill, if ever it be given to me in the course of that career, to do anything for the promotion of the science which they taught me to love.

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CHAPTER I.

The instrument employed was the heliometer belonging to the Cape Observatory, which is fully described in Vol. VII of the Cape Annals.

The observations were made by Dr. (now Sir DAVID) GILL and on a few occasions by Mr. W. H. FINLAY.

For the purpose of determining the scale-value of the heliometer and the index-error of the position-circle, a pair of standard-stars was measured on each occasion when the satellites were observed.

In Table I are given the dates on which observations were secured. The different columns contain the rotation-numbers of the observations indicated by the headings; *p* denotes an observation of position-angle and *s* of distance. Rejected observations are included in brackets.

N.B. To avoid confusion and to facilitate reference to the details of the observations, which will be published in the Cape Annals, the original rotation-numbers have been retained through the whole work.

TABLE I. OBSERVATIONS OF JUPITER'S SATELLITES.

Date	Obsr	Victoria-standards		Standards		I—II		I—III		I—IV		II—III		II—IV		III—IV	
		p	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*
1891																	
Aug. 17	G					4	3	9	10	2	1	11	12	5	6	7	8
19	G			13	14	19	20	25	26	17	18	21	22	23	24	15	16
20	G			31, 41	32, 42	33	34	27	28	37	38	35	36	29	30	39	40
21	G			47	48	43, 49	44, 50									45	46
24	G			55, 65	56, 66	63	64	61	62	59	60	51	52	57	58	53	54
25	G			67, 75	68, 76	79	80, 81	71	72	82	83	73	74	77	78	69	70
27	F			90	91					85	84			89	88	[87]	86
29	G		107	98	99, 106	102	103	100	101	92	93	104	105	94	95	96	97
31	G		119	110, 116	111, 117, 118	120	121	122	123	124	125	108	109	114	115	112	113
Sept. 1	G		136	128, 134	129, 135	141	[142]	132	133	130	[131]	139	140	137	138	126	127
5	G	143	[160]	148, 152	149, 159	144	145	155	156	150	151	153	154	157	158	146	147
7	G				173	163	164	165	166	161	[162]	171	172	167	168	169	170
8	G		184	179	180, 183							174	175	176	177	178, 182	181

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TABLE I. OBSERVATIONS OF JUPITER'S SATELLITES — Continued.

Date	Obsr	Victoria-standards		Standards		I—II		I—III		I—IV		II—III		II—IV		III—IV	
		p	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*
1891					190												
Sept. 10	G		200	197	198 199	193	194	187	188	195	196	185	186	191	192		189
13	G			207	208	203	204	205	206	213	214	201	202	211	212	209	210
14	G	215	233	31	[228] 232	220	221	218	219	226	227	216	217	222	223 230	224	225 229
18	F			246	247	234	[235]	240	241	[238]	[239]	242	243	[236]	[237]	244	245
19	G		265	255 262	254 263 264	248	249	250	251	258	259	252	253	260	261	256	257
20	G		282	272 280	273 [281]	268	269	270	271	266	267	278	279	276	277	274	275
21	G			285 [297]	286 298	293	294	291	292	295	296	[283]	284	287	288	289	290
22	G		315	307 313	308 314	299	300	301	302	309	310	303	304	311	312	305	306
23	G		327	316 325	317 326							320	321	322	323	318 324	319
24	G					328	329										
28	G			330	331	332	333										
29	G			338	339	334	335	340	341							336	337
Oct. 4	G			348	349	346	347	354	355	344	345	342	343	352	353	350	351
6	F			362	363			364	365	360	361			358	359	356	357
9	G			372	373	368	369	366	367	376	377	378	[379]	374	375	370	371
10	G							380	381							382	383
13	G			390	391	384	385	388	389	397	398	386	387	395	396	393	394
14	F			411	412	399	400	405	406	403	404	409	410	401	402	407	408
15	F											417	418	415	416	413	414
16	F			427	428	423	424	429	430	425	426	421	422	419	420	431	432
17	F			439	440			433	434	435	436					437	438
19	F			447	448	445	446	[441]	442	449	450	443	444	453	454	451	452
23	F			461	462							457	458	455	456	[459]	[460]
Nov. 5	G			469	470	465	466	463	464	473	474	[467]	[468]	471	472	475	476
6	G			483	484	481	482	479	480	485	486	[477]	478			487	488
8	G			493	494							489	490	491	492	495	496
9	G					497	498										499
19	G															[500]	[501]
22	G			508	509	502	503				512	504	505	510	511	506	507
27	G			519	520			515	516	513	514					517	518
30	F					[521]	[522]	[523]	[524]								
Dec. 8	F			531	532	527	528	525	526	529	530	537	538	535	536	533	534
14	G			545	546			549	550	547	548	[541]	[542]	543	544	539	540
Total number of Observations						34	33	32	33	31	30	30	31	34	35	38	44

A complete observation of a pair of satellites consists of the following operations:

1° one pointing in position-angle, and turn reversing prism through 90°,

2° one pointing in position-angle,

3° one pointing in distance, and turn prism,

4° one pointing in distance,

5°—8° exchange the semilenses and repeat the same operations in the reverse order.

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In this way the mean epoch of the distance and position-angle measures will be approximately the same.

In the observations of standard-stars generally only one pointing in position-angle and one in distance was made in each position of the semilenses.

Sometimes it has occurred, when two pairs of satellites were in approximately the same distance, that after exchanging the semilenses another pair was observed than before. This is the case with the observations 236—237, 238—239, 467—468, 541—542. These observations must of course be rejected.

On a few occasions, when two satellites were very near together, the measures of distance and position-angle were combined by placing the images at half or double distance, as is done in measuring a double star.

Sometimes four pointings were made instead of two in each position of the semilenses.

The instant a pointing is made the observer calls „stop" and an assistant notes the time from a chronometer to the nearest second. The mean of the times of the four pointings, of which an observation consists, was corrected for chronometer-error and reduced to Greenwich mean time. The adopted longitude of the Cape Observatory is $1^h13^m54^s.8$ East of Greenwich. The corrections for aberration-time were applied to these epochs using the distance of Jupiter taken from the Nautical Almanac [1]) and adopting for the light-equation $498^s.46$. The times were computed to a tenth of a second and the resulting times were rounded off to hundredths of a minute. These times, together with the observed distances and position-angles, are given in tables IV—XV.

For computing the hour-angles and zenith-distances, which determine the refractions, the place of Jupiter taken from the Nautical Almanac was used. The observations of distance were reduced with a numerical accuracy of $0''.001$, while the position-angles were computed to $1''$, which corresponds to $0''.001$ of a great circle for a distance of $206''$.

The observations were not reduced to the centre of the earth.

The corrections applicable on this account are

$$\frac{\delta s}{s} = -\pi\,\rho\cos\zeta$$

$$\delta p = \pi\rho\cos\varphi'\tan\delta\sin t$$

where

π = equatorial horizontal parallax

ρ = radius-vector, φ' = reduced latitude } of the observer

δ = declination, ζ = zenith-distance, t = hour-angle } of the middle point

If we take $\pi = 2''.1$, $\varphi' = -34°$, $\delta = -9°$, $\rho = 1$

we get the following corrections:

hour-angle	$\pm 1^h$	$\pm 2^h$	$\pm 3^h$	$\pm 4^h$
$\frac{\delta s}{s}$	—0.0000088	—0.0000081	—0.0000068	—0.0000051
δp	$\mp\ 0''.1$	$\mp\ 0''.1$	$\mp\ 0''.2$	$\mp\ 0''.2$

which can safely be neglected. In the case of the distances a very small systematic error can thus be introduced. We will return to this point later on (Chapter III).

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[1]) See however page 18.

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Full details of the observations and reductions will be published in Vol. VIII of the Annals of the Cape Observatory.

The heliometer cannot give *absolute* determinations of distance and position-angle, because the scale-value, the index-error of the position-circle, and the equatorial adjustment, alter with the temperature of the different parts of the instrument. A pair of stars was therefore selected near the place of Jupiter and approximately in the position-angle of the plane of the satellites, and this standard pair was observed on every night when observations of the satellites were made.

The standard-stars are

S.D. —8°.5905 mag. 7.0 and S.D. —7°.5838 mag. 6.2.

They were compared by the heliometer with the standard-stars of the triangulation of the Victoria Comparison-stars, of which the distance and position-angle are probably better known than of any other pair of stars in the heavens. This comparison gave as the mean of 9 observations in distance and 3 observations in position-angle (equinox 1891.0; epoch 1891.75):

$$s = 6705''.36 \pm 0''.065 \qquad p = 68° 4' 30'' \pm 2''.1$$

[The probable errors include those of the adopted distance and position-angle of the Victoria standard-stars.]

The standard-stars were also observed on the meridian, each on 15 nights. The results of these observations are published in the Cape Catalogue for 1890. From these places we derive (for the same epoch and equinox):

$$s = 6705''.29 \qquad p = 68° 4' 32''.$$

The comparison of the two values shows how accurately the position-angle and distance of the standard-stars are determined. The first mentioned values have been adopted.

The absolute scale-value is therefore certainly known within $\frac{1}{60000}$ part. If there is any systematic error in the scale-value, depending on the position-angle and not eliminated by the use of the reversing prism — which is very improbable — or if there is any error in the adopted refraction constants, such errors will be eliminated, owing to the circumstance that the standard-stars are in approximately the same position-angle as the satellites, and have been observed at approximately the same hour-angle. We would consequently (if the accidental errors of observation were small enough) be able to determine the mass of Jupiter within $\frac{1}{20000}$ of its amount, *provided all heliometer measures were free from systematic errors depending on the distance*, or in other words: if the scale value was the same for all distances.

This is however not the case.

In the Cape heliometer, as in all modern heliometers, there is in the focal plane of the eyepiece a square formed by flat wires, adjusted parallel and perpendicular to the direction of motion of the semilenses. This system of wires rotates with the instrument and becomes the means by which directions are estimated in the illuminated field. Before observations are commenced the reversing prism is turned so as to make the apparent direction of these wires vertical and horizontal.

Since the superposition of two images is not an accurate method of measuring, every heliometer observer tries to make the images of the two stars pass and repass trough each other by a small motion in position-angle $\pm \psi$. (The equality of the two angles $+\psi$ and $-\psi$ is easily estimated in terms of the diameter of the star discs.) In causing the images to „swing through" he instinctively uses the pair of lines at right angles to the distance-motion as a guide. He adjusts the distance till the stars appear as represented in fig. 1, and then immediately moves the instrument in position-angle, wenn the stars will appear as in fig. 2. When he has satisfied himself that the line joining the images is parallel to the wires p in both positions, he gives the signal „stop" and proceeds to read the scales.

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The most refined judgment of this parallelism with the wires is inseparable from the plan of rapidly „crossing" (that is exchanging the position of) the images. The eye in this operation is extremely sensitive

Fig. 1.

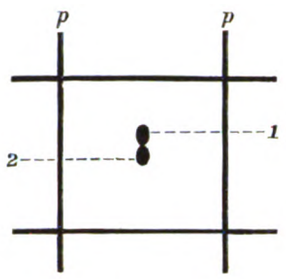
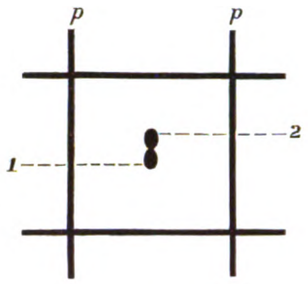


Fig. 2.



to a minute change in the apparent position-angle, and the method is probably the most refined that we possess as a practical means of measuring the angular distance between optical discs with high precision. The optical discs formed by the half lenses are elongated in the direction perpendicular to the diameter along which the lens has been cut, that is parallel to the wires *p*. This circumstance greatly facilitates the judgment of parallelism.

Figs 3 and 4 represent the positions of the images in the focal plane. The angle ψ is of course greatly exaggerated, as is also the distance between the two wires *p*.

Fig. 3.

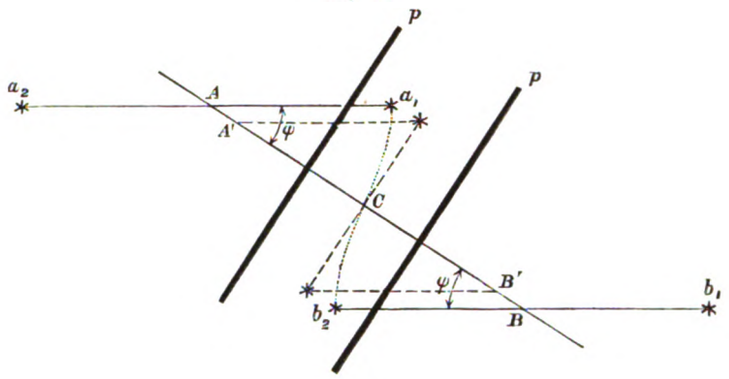
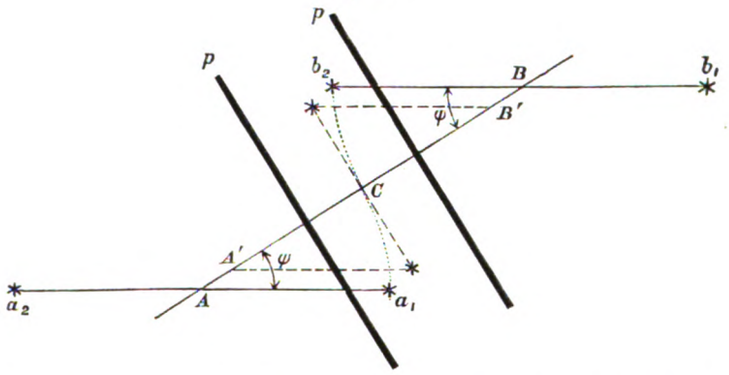


Fig. 4.



A and B are the projections on the focal plane of the optical centres of the two semilenses. The line AB shews the direction of motion of the slides. C is the intersection of the central axis of the instrument with the focal plane. The optical axis of lens A is supposed to be pointing to star 1 and that of lens B to star 2. Lens A gives images of the two stars at a_1 and a_2 and lens B at b_1 and b_2. The direction of the lines $a_2 a_1$ and $b_2 b_1$ represents thus the true position-angle of the stars in the heavens. In the figures the separation of the lenses is supposed to correspond to the exact distance between the stars, so that by a rotation in position-angle the two images a_1 and b_2 can be brought to coincidence in C. The observer does

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however not try to effect that coincidence, but, while the line of motion makes an angle $\pm \psi$ with the true position-angle, he tries to make the line joining the images a_1 and b_2 parallel to the wires p. To do this he must diminish the separation of the optical centres A and B till they occupy the positions A′ and B′. The measured distance is thus not s, but $A'B' = s \cos \psi$, and the correction to be applied to the measured distance is

$$\Delta s = + s \sin^2 \tfrac{1}{2} \psi = + \frac{\varkappa^2}{2s},$$

where $\varkappa$ is the distance of the artificial double star formed by the images a_1 and b_2 [1]).

Before and after crossing through, that is in the two positions represented by *figs* 3 and 4, there is a small difference in the direction of the line of motion of the slides. The observer is not aware of this, as the wires p, by which he can only be guided in estimating directions, *rotate with the instrument* and remain always perpendicular to the line AB. If the distance is so short, that the change in direction of the wires p, caused by the small motion in position-angle needed to cross the images, becomes noticeable, a better method of observing is *not* to cross through, but to make one pointing as represented in *fig.* 3, but with the images just touching each other (as in *fig.* 1), so that the major axes of the optical ellipses, which are always parallel to the wires p, are in one line. Then, after rotating the reversing prism through 90°, the next pointing is made as in *fig.* 4 (or 2). The distance actually measured is again $s \cos \psi$, and needs the correction $+ \frac{\varkappa^2}{2s}$. This plan was adoptod by Dr. GILL in all the observations of the satellites.

For all distances under 100″ with a few exceptions, and for one distance of 145″, the angle ψ actually used has been recorded; that is to say the images were placed as in *fig.* 1 and distance *and position-angle* read off; then for the next pointing the images were placed as in *fig.* 2 and the distance and position-angle were again read.

The difference between the two position-angles, corrected for the motion of the satellites in the interval, is 2ψ. These distances have of course been corrected accordingly, and are thus free from personal error.

From these observed values of ψ we find the following values of $\varkappa$ for the two observers.

TABLE II. $\varkappa$ FROM OBSERVED ψ.

GILL										FINLAY		
s	$\varkappa$	Defn	Mean s	Mean $\varkappa$	s	$\varkappa$	Defn	Mean s	Mean $\varkappa$	s	$\varkappa$	Defn
"	"		"	"	"	"		"	"	"	"	
22.0	3.71	2	22.3	2.89	44.0	4.36	2-3	50.1	2.72	35.2	7.18	2-3
22.4	2.40	2-3			44.1	2.09	2			56.0	5.01	2-3
22.5	2.56	2-3			44.2	2.01	2			71.3	7.49	3
					49.9	2.80				83.5	6.23	2-3
30.4	3.65	3-4	34.9	3.23	55.9	2.09	1 2			145.1	7.44	2-3
31.2	3.08	2-3			56.0	3.04	1-2					
33.0	2.69	2			56.9	2.65	1-2			*Mean* $\varkappa = 6.''67$		
34.9	2.90	2			80.5	2.67	2-3					
38.6	3.16	2-3			82.2	3.54	3	85.5	3.08			
39.0	3.30	2-3			83.2	3.61	3					
39.4	3.81	3			88.7	3.77						
					92.7	3.02	3					
Mean of all observations: $\varkappa = 3.''04$												

1) The whole of this explanation has been copied almost verbatim from Cape Annals, Vol VI. part 1, pages 234 sqq.

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It appears that GILL's $\varkappa$ is very small indeed and very constant. FINLAY's observations are too few to permit very definite conclusions to be drawn.

If we arrange GILL's observations according to the estimated quality of images we get the following means:

Defn 1 and 1-2	$\varkappa$ = 2".59	3 obsns
2 „ 2-3	2.91	12 „
3 „ 3-4	3.55	5 „

The influence of the sharpness of the images is very marked. It seems to be impossible with images 3 or 3-4 to bring the discs closer together than 3".

It is a very important result that $\varkappa$ shews no sign of variation with the distance. If this is the case with distances up to 1000" we may be tolerably certain that after correction for $\varkappa$ all distances are free from systematic error.

As the observer was constantly conscious of the necessity of keeping the amount of „crossing" as nearly constant as possible, it is very probable that in the ordinary observations, by crossing through without measuring ψ, $\varkappa$ is even more constant, and perhaps also smaller, than in these exceptional cases, where this necessity does not exist, since the value of ψ is recorded and the correction due to the amount of crossing can afterwards be applied.

The value of $\varkappa$ for average definition must be determined for each observer from the observations themselves, by introducing a corresponding unknown quantity into the equations of condition. It must be assumed, that the quality of definition is in the mean the same for all distances. That this assumption is not very far from the truth, will be seen from the following little table containing the number of observations of different distances, made with different qualities of images.

GILL

Distances	Nr of Obsns in Defn			Mean defn
	1&1 2	2&2-2	3&3-4	
" "				
100—200	5	19	2	2.1
200—400	9	30	.9	2.2
400—1000	6	33	11	2.4

FINLAY

Distances	Nr of Obsns in Defn			Mean defn
	1&1-2	2&2-3	3&3-4	
" "				
100—200	0	8	2	2.3
200—400	0	7	4	2.6
400—1000	0	8	7	2.6

Observations in which ψ was recorded, rejected observations, and observations in which there is no note about the quality of the images are excluded from this table. Whether the observations corrected for $\varkappa$ are really free from systematic error, will appear from the final residual errors.

For the different values of $\varkappa$ we have the following corrections to different distances.

$\varkappa$ \ s	50"	100"	200"	500"	1000"
"	"	"	"	"	"
2.5	0.062	0.031	0.016	0.006	0.003
3.0	0.090	0.045	0.022	0.009	0.004
3.5	0.122	0.061	0.031	0.012	0.006
6.7	0.449	0.224	0.212	0.045	0.022

It will be seen that with GILL's $\varkappa$ for average definition the correction for 100" (the smallest distance observed without recording ψ) is only $\frac{1}{2000}$ of the distance, and for the average distance of 300" it

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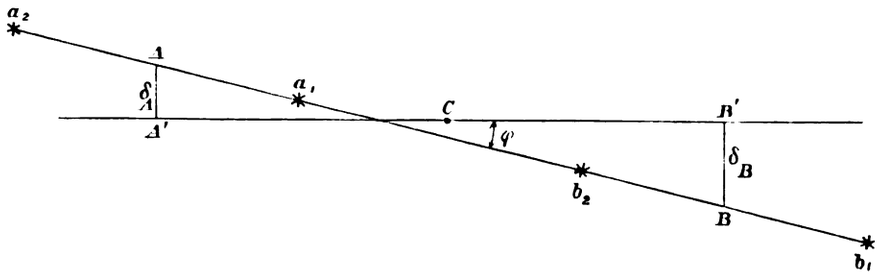
is only $\frac{1}{20000}$ of the distance. It will therefore be sufficient if the correction due to $\varkappa$ can be determined within $\frac{1}{3}$ of its amount in order that the distances may in the mean, apart from errors of observation, be reliable within $\frac{1}{60000}$ and consequently the mass of Jupiter within $\frac{1}{20000}$ of its amount.

There is still another source of error, which however only affects the distances measured by placing the images at half or double distance. The optical centres of the two semilenses have so far been supposed to be in the plane of motion, of which the intersection with the focal plane is represented by the line AB in *fig.* 3. This will however generally not be the case. Let the distances of the optical centres of the semilenses A and B from that plane be δ_A and δ_B, then, when the lenses are crossed, the optical centres never come into exact coincidence, but pass clear of each other at a minimum distance $\delta = \delta_A + \delta_B$ (It may be remarked that δ can be both positive or negative. The two semilenses are made by cutting one complete lens in two pieces, and these may be afterwards brought closer together than the thickness of the sawcut, which has been taken out).

Fig. 5 represents a measure of double distance.

Fig. 5.



The four images are visible in the field at the same time, and are placed so that they lie in a straight line, and the distances $a_2 a_1$, $a_1 b_2$ and $b_2 b_1$ are equal. It is then evident from the diagram that the separation of the lenses A′B′ is not $2s$, but $2s \cos \varphi$, while φ is determined by the relation

$$\sin \varphi = \frac{\delta_A + \delta_B}{2s}.$$

The measured position-angle will be $p + \varphi$ instead of p.

On reversing the lenses, A′ and B′ change places, (A and B moving parallel to the lines A′B′) and the measured distance will again be $2\ s \cos \varphi$, but now the measured position-angle is $p - \varphi$.

The position-angle is therefore correct in the mean, and the difference between the two measured values affords a means of determining φ and hence $\delta = \delta_A + \delta_B$.

The same reasoning applies of course to measures of half distance, when the images are placed so that b_2 comes midway between a_2 and a_1, and a_1 midway b_2 and b_1.

The distances measured in this way have been corrected with the value of φ derived from the position-angles.

From the observations of the position-angles of the standard-stars it appears that throughout the period of observation δ has varied between narrow limits, the mean value being $+ 1''$.

It is not impossible that, owing to a slight curvature of the slides, δ has different values for different distances.

In ordinary observations made by the method of crossing through the influence of this distance of optical centres disappears.

Figs 6 and 7 represent the positions of the images in such an observation, as they appear in the field when the apparent direction of the wires p has been made vertical by the use of the reversing prism. The meaning of the different symbols is the same as in the preceding figures. One pointing is represented

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by *fig.* 6; the measured distance is $A'B' = s \cos(\psi - \Phi)$. Then, after crossing through, a second pointing is made as in *fig.* 7 and now the measured distance is $s \cos(\psi + \Phi)$. When ψ and Φ are both small the mean will be $s \cos \psi$, and therefore independent of Φ, that is of the distance of optical centres [1]).

Fig. 6.

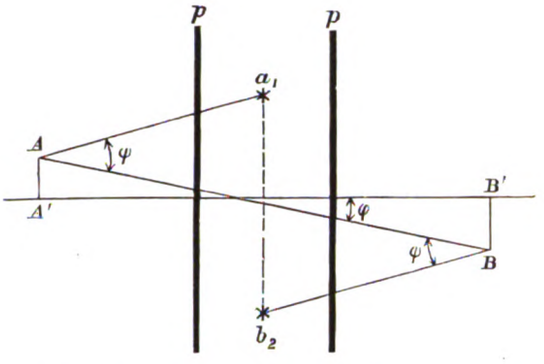
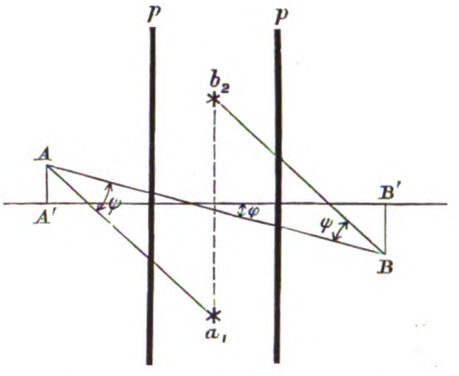


Fig. 7.



For long distances (over 1000″) the most accurate method of observing is by crossing through. If the distance of optical centres is not zero the line of the images, if parallel to the wires p, will be no longer so after crossing through. The difference will however be extremely small and scarcely, if at all, noticeable. The observer while crossing through finds himself unable to satisfy the condition of exact parallelism in both positions (*figs* 1 and 2) by the same adjustment of the separation of the lenses, and will instinctively make a compromise between the two adjustments which would produce exact parallelism in the two cases. Thus his measure practically represents the mean of the two measures illustrated by *figs* 6 and 7, and the only effect of the distance of optical centres will be to make the observations perhaps slightly more difficult.

As is already explained, in observing the satellites the two pointings represented by *figs* 6 and 7, corresponding to *figs* 1 and 2, were always made separately without crossing through (at least certainly so by Gill, and probably also by Finlay). The mean can therefore not be affected by the distance of optical centres. In observing distances below 100″ the angle ψ was moreover recorded, as is explained above.

For distances under 20″ the most reliable method is to put the images at half distance, and for distances under 5″ at double distance, provided Φ or δ be determined by simultaneous observations of position-angle. This method was always employed for very short distances, with one exception, when Mr. Finlay measured a distance of 6″ by crossing through. This observation has been rejected.

The general conclusion is thus that the distance of optical centres affects only those distances for which its influence can be rigorously determined from the observations themselves, and cannot introduce any systematic error.

In the tables IV—XV the column headed "method" gives the method of observing which was used; c means the ordinary method of crossing through without recording ψ, ψ shows that the position-circle was read to determine the angle $\pm \psi$, d and h denote that the images were placed at double, respectively half distance.

The position-angles are not supposed to be affected by systematic errors. It has already been shewn that the distance of optical centres does not affect the mean of the measures in the two positions of the lenses. The crossing through, which is here done by rapid and alternately reversed motion in distance, does not introduce any error at all. We must further assume that there is no personal error. The personal

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1) It may be remarked that the angles φ in *figs* 6 and 7 are not exactly equal, but the difference is a small quantity of the second order, if φ itself is of the first order.

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difference of 6″ between GILL and FINLAY which was found from the Victoria-triangulation work (*Cape Annals*, *Vol* VI, *part* 1, *page* 173) is absolutely negligible for the present purpose. Mr. FINLAY's observations are too few to allow a determination of personal errors to be made from the satellite observations themselves. If there was a systematic error depending on the position-angle, this would be eliminated by the determination of the instantaneous index-error from the observations of the standard stars, which are in approximately the same position-angle as the satellites. A systematic error depending on the distance is very improbable a priori, and it will be seen from the residuals (Chapter III, table XVIB) that no error of this kind does exist.

The corrections for equatorial adjustment have been applied using the constants given in ***Cape Annals***, *Vol* VI, *part* 1, *page* 162 [1]). The instantaneous index-error was determined every night by observations of the standard stars, either at the beginning and end, or in the middle of the night's work. As these stars were very near Jupiter, the index-error thus determined includes the effect of deviations from the adopted equatorial adjustment at the mean hour-angle of the night.

In the comparison of the standard stars with the Victoria standards the two pairs were also observed at the same hour-angle. It has already been pointed out that the adopted position-angle of the standard stars derived from this comparison can be considered correct to a few seconds. If there are any systematic errors in the adopted refraction-constants, such errors will be eliminated by the use of the index-error derived from the standard-stars owing to the circumstance that these stars are in approximately the same position-angle as the satellites. The measured position-angles of the satellites, when corrected for equatorial adjustment and instantaneous index-error, therefore represent, within the errors of observation, absolute position-angles referred to the true pole of the heavens at the epoch of observation.

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1) For errata in the formula there given see *Cape Annals*, vol VIII, Part 2, page 115 B. On the same page (of Vol VI), last line, for — 26″.1 read + 26″.1.

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CHAPTER II.

In order to derive from the observations corrections to the fundamental constants of SOUILLART's theory, the distances and position-angles for the epochs of observation must be computed from that theory and compared with the observed values.

This part of the work has been much simplified by the use of Mr. MARTH's *Data for computing the positions of Jupiter's Satellites*, published in Vol. 51 of the *Monthly Notices of the R. A. Society*, pages 505—539.

The positions of the satellites are there referred to a fundamental plane, which is the adopted plane of Jupiter's equator. It must te remarked that, in this investigation, this plane must only be considered as an arbitrarily adopted plane, so chosen as to facilitate the computations, and which is defined by its adopted node and inclination in reference to the plane parallel to the earth's true equator. The hypothesis that this plane coincides with Jupiter's equator is not made use of, nor is it possible to determine the position of that equator from observations at one epoch only. The zero of longitudes in this plane is its descending node on the plane of Jupiter's orbit, *i. e.* Jupiter's vernal equinox.

Marth's data and tables were very carefully examined and compared with Souillart's theory, on which they profess to be based, and in addition to a few unimportant errors in the last decimal places, the following discrepancies were found.

Of Marth's perijovia, given on page 511, ϖ_2 and ϖ_4 agree with Souillart's $g_1 t + \beta_1 - b_4 t - \gamma_4 + 180^\circ$, resp. $g_3 t + \beta_3 - b_4 t - \gamma_4 + 180^\circ$, but ϖ_1 and ϖ_3 differ from Souillart's values by $+ 20^\circ.0022$ and $+ 21^\circ.0000$ (Marth—Souillart). The motions of all the perijovia agree with Souillart. As the coefficients of the inequalities do not depend on the adopted longitudes of the perijovia, it is not of the least importance which values are adopted. The values printed on page 511 have actually been used in the formation of the arguments, and in the computation of the inequalities S on page 512.

The values of G_2 given on page 512 are in error by a constant quantity $- 1^\circ.9755$ (Marth—Souillart). The *correct* values (*i. e.* those given by Marth $+ 1^\circ.9755$) have been used in the inequalities S and in the formulae on page 513.

The coefficients of the inequalities are all in agreement with those given by Souillart. A curious mistake has been made by Marth in the tables of the second satellite. On page 514 he gives the inequalities:

Radius Vector.		Longitude.	
$- \cdot0000672 \cos \beta_2$		$+ 48''.578 \sin \beta_2$	
$+ \quad 634 \cos 2\beta_2$	β_2	$- 20.498 \sin 2\beta_2$	β_2
$+ \quad 061 \cos 3\beta_2$		$- \quad 1.505 \sin 3\beta_2$	
$+ \quad 770$			

In these formulae $2\beta_2$ must be replaced by $3\beta_2$ and $3\beta_2$ by $5\beta_2$. In the table for the longitudes, page 531, the *correct* arguments ($3\beta_2$ and $5\beta_2$) have been used, but the table for the radius vector (pages

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530—531) is computed with the *erroneous* arguments ($2\,\beta_2$ and $3\,\beta_2$). Thus the radii vectores, computed from Marth's tables require a correction:

$$\delta\rho_2 = +\ 0.0000634\ (\cos 3\beta_2 - \cos 2\beta_2) + 0.0000061\ (\cos 5\beta_2 - \cos 3\beta_2).$$

In addition to these errors the following errata were noticed:

page 510	line 6	for $65°.073$ . .	read $65^{s}.073$
513	8	sin V	cos V
	31	$a_2 = l_1 - l_2$ . .	$a_2 = l_1 - l_2 + 180°$
514 First Satellite, Radius Vector		$-\cdot 0000607$ . .	$-\cdot 0000703$
515 Fourth Satellite, Longitude		$2.208 \sin \varepsilon_4$ . .	$4.208 \sin \varepsilon_4$
516 column: log a_2 argument 7.8		794 . .	774
526 column a_1, argument $\cdot 190$		8458 . .	8558
	$\cdot 310$	1328 . .	1338
529 column a_2, argument 8^{m}		$\cdot 00148$ . .	$\cdot 00158$
531 Longitude (continued), column γ_2, argument .05		$\mp\ \cdot 0039$ . .	$\pm\ \cdot 0039$
539 Longitude (continued), Heading		$\eta_4\beta_4$	$\eta_4\delta_4$

The longitudes and radii vectores were interpolated from Marth's tables for the epochs of observation in distance; the tabular distances and position-angles were then computed by the following formulae:

$$\Delta' = \Delta + a\rho \cos(v - L) \cos B.$$

(1)
$$x'' = \frac{a''\rho}{\Delta'} [\sin(v - L) - \sin(v - \Omega) \sin i \tan \tfrac{1}{2} i \cos (L - \Omega)]$$

$$y'' = \frac{a''\rho}{\Delta'} [\cos(v - L) \sin B + \sin(v - \Omega) \sin i \cos B]$$

where

a = mean distance
ρ = radius vector
v = true longitude
i = inclination
Ω = ascending node

} of the satellite

$L + 180°$ = jovicentric longitude
B = „ latitude
Δ = „ distance

} of the earth

$$a'' = \frac{a}{\sin 1''}.$$

The quantities belonging to the satellites I, II, III, IV will be distinguished by the suffixes 1, 2, 3, 4.

x'' and y'' are then the coordinates, expressed in seconds of arc, of the projections of the satellites on a sphere through Jupiter's centre, the axes of x'' and y'' being perpendicular and parallel to the projection on that sphere of a line perpendicular to the fundamental plane.

Δ is the distance from the position of the earth at the actual instant of observation to the point where Jupiter was, when the observed rays left it. The Nautical Almanac gives the distance from the earth at Greenwich noon to Jupiter at Greenwich noon. To the log Δ interpolated from the N.A. a correction was therefore applied, which is [1])

$$\delta(\log \Delta) = M\nu R \sin(\Delta - L)$$

---

1) This correction as given by Marth in his *Ephemeris for physical observations of Jupiter* M.N. Vol 51, page 360, is erroneous, as was pointed out by Marth himself on page 575 of the same volume.

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where: M = the modulus of the system of logarithms = 0.434

ν = the heliocentric motion of Jupiter (expressed in parts of the radius) in the light-time (498^s)

R = Jupiter's radius vector

$\Lambda + 180°$ = the sun's jovicentric longitude.

A similar correction, *viz:*

$$\delta L = - \nu R \cos (\Lambda - L)$$

ought to have been applied to the values of L. This correction however never amounts to 0".00024 throughout the time of observations and has been neglected.

The unit of Δ and of a is the mean distance of the earth from the sun.

The second term in the expression for x'' has been neglected for satellites I and III.

From the coordinates the distances and position-angles were computed by the formulae:

(2)
$$s \sin (p - \mathrm{P}) = x_2'' - x_1''$$
$$s \cos (p - \mathrm{P}) = y_2'' - y_1''$$

where $p - \mathrm{P}$ is the position-angle reckoned in the usual direction from the direction of the y''-axis, that is from the adopted position-angle of Jupiter's axis of rotation.

The position-angles so computed are those for the epochs of the observations in distance. They were reduced to the instants of the corresponding observations in position-angle by means of the differential coefficients $\frac{dp}{dt}$. In consequence of the symmetrical arrangement of the observations, the difference between these two epochs is seldom more than a minute, and it is sufficiently exact to take

$$\frac{dp}{dt} = n_1 \frac{dp}{dv_1} + n_2 \frac{dp}{dv_2},$$

where n_1 and n_2 are the mean motions in longitude.

When two satellites are close together and crossing each other at a short distance, so that the change in distance and position-angle is rapid and cannot be assumed proportional to the time, the coordinates x'' and y'' were computed separately for the epochs of each pointing.

In this computation however the value of the factor $\frac{a''\rho}{\Delta}$, which changes very slowly, was always taken for the mean epoch. The longitudes, taken from Marth's tables for the mean epoch were reduced to the epochs of the separate pointings by means of corrections $\Delta v = n\,\Delta t$. The coordinates x'', y'' and the position-angles $p - \mathrm{P}$ computed from them are thus not rigorously the true values for the separate epochs, but the mean of the separate position-angles will represent the true position-angle at the mean epoch with all necessary accuracy.

Throughout the computation of tabular distances and position-angles 6-figure logarithms were used. All the computations were carefully examined by Mr. A. Cochrane, one of the most experienced computers of the Cape Observatory. Moreover the longitudes, radii vectores and rectangular coordinates were checked by an independent computation of the differential coefficients $\frac{dv}{dt}$, $\frac{ds}{dt}$, $\frac{dx''}{dt}$, $\frac{dy''}{dt}$, and of the differences of these quantities in different observations on the same date by the formulae

$$\Delta v = \frac{dv}{dt} \Delta t \ldots . \text{ etc.}$$

The tabular distances and position-angles were checked by recomputing from them the differences of rectangular coordinates ($x_2'' - x_1''$ and $y_2'' - y_1''$).

A slight correction must still be applied to these tabular places on account of the error in Marth's

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table of the inequality depending on β_2, which was mentioned above. This error was only discovered after the computation of the tabular places. An auxiliary table was then prepared of the correction

$$\delta\rho_2 = + 0.0000634 (\cos 3\beta_2 - \cos 2\beta_2) + 0.0000061 (\cos 5\beta_2 - \cos 3\beta_2).$$

and $\delta\rho_2$ was taken from this table for every observation. A correction

$$\delta s = \frac{\partial s}{\partial \rho_2} \delta\rho_2$$

was then added to the computed distances.

The effect of this error on the position-angles has been neglected, as it exceeded 0".003 of a great circle in only two cases, and in both of these it was smaller than 0".010.

The tabular distances thus computed are those between the projections of the positions of the satellites at the epoch of observation, corrected for aberration using the distance of Jupiter from the earth, that is the positions where the satellites were at the time when the observed rays of light left *Jupiter*. But what is observed is the distance between the projections of the positions of the satellites at the time when the observed rays left the *satellites*, that is at two different epochs for the two satellites of a pair. We must thus apply to the tabular distances a further correction, which is computed by the following formulae (The same reasoning holds of course for the position-angles).

$$\delta s = \frac{\partial s}{\partial r_1} \delta r_1 + \frac{\partial s}{\partial r_2} \delta r_2,$$
$$\delta r = n \, \delta t,$$
$$\delta t = (1 - r) \times 498^s.46.$$

This correction was at first overlooked. The values of O — C, which are given in the tables IV—XV at the end of this chapter do *not* include it. The columns headed „differential aberration" give then the correction which is to be added to O — C, *i. e.* — δs (respectively — $s \sin 1'' \, \delta p$). It will be seen that for the position-angles its influence is absolutely negligible. For the distances the corrections are exceedingly small, and would also be negligible, if they were not systematic. They are systematic however and have been taken into account.

Subtracting the tabular from the observed quantities we find the absolute terms (O — C) of the equations of condition. For the position-angles these were multiplied by $s \sin 1''$ to reduce them to seconds of arc of a great circle.

To form the left hand side of the equations we have now to compute the differential coefficients

$$\begin{array}{ccccc} \frac{\partial s}{\partial a_i} & \frac{\partial s}{\partial r_i} & \frac{\partial s}{\partial \rho_i} & \frac{\partial s}{\partial p_i} & \frac{\partial s}{\partial q_i} \\ \frac{\partial p}{\partial a_i} & \frac{\partial p}{\partial r_i} & \frac{\partial p}{\partial \rho_i} & \frac{\partial p}{\partial p_i} & \frac{\partial p}{\partial q_i} \end{array}$$

where

$$\begin{aligned} p_i &= i_i \sin(-\Omega_i) \\ q_i &= i_i \cos(-\Omega_i) \end{aligned} \qquad (i = 1, 2, 3, 4).$$

(The minus sign is introduced in comformity with Marth's notation).

As the observations extend over only four months, the mean motions must be assumed known. This enables us by Kepler's third law to introduce a single unknown quantity $\mu = \frac{\text{mass of sun}}{\text{mass of Jupiter's system}}$ instead of the four a's

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If v, p and q are expressed in degrees, and if we put

$$X_i = \frac{a_i'' \rho_i}{\Delta_i} \sin 1° \cos (v_i - L)$$

$$Y_i = \frac{a_i'' \rho_i}{\Delta_i} \sin 1° \{ - \sin (v_i - L) \sin B + \sin i_i \cos (v_i - \Omega_i) \cos B \}$$

$(i = 1, 2, 3, 4)$

we find:

$$\rho_1 \frac{\partial s}{\partial \rho_1} = - x_1'' \sin (p - P) - y_1'' \cos (p - P) \qquad \frac{\partial s}{\partial v_1} = - X_1 \sin (p - P) - Y_1 \cos (p - P)$$

(3)

$$\rho_2 \frac{\partial s}{\partial \rho_2} = x_2'' \sin (p - P) + y_2'' \cos (p - P) \qquad \frac{\partial s}{\partial v_2} = X_2 \sin (p - P) + Y_2 \cos (p - P)$$

$$\rho_1 \frac{\partial s}{\partial \rho_1} + \rho_2 \frac{\partial s}{\partial \rho_2} = s$$

$$s \sin 1'' \rho_1 \frac{\partial p}{\partial \rho_1} = - x_1'' \cos (p - P) + y_1'' \sin (p - P) \qquad s \sin 1'' \frac{\partial p}{\partial v_1} = - X_1 \cos (p - P) + Y_1 \sin (p - P)$$

(4)

$$s \sin 1'' \rho_2 \frac{\partial p}{\partial \rho_2} = x_2'' \cos (p - P) - y_2'' \sin (p - P) \qquad s \sin 1'' \frac{\partial p}{\partial v_2} = X_2 \cos (p - P) - Y_2 \sin (p - P)$$

$$\rho_1 \frac{\partial p}{\partial \rho_1} + \rho_2 \frac{\partial p}{\partial \rho_2} = 0$$

Kepler's third law gives

$$\frac{d \odot \mathcal{M}}{\odot \mathcal{M}} = - 3 \frac{da_i}{a_i} = - 3 \frac{d\rho_i}{\rho_i}$$

from which we find

(5)

$$\frac{\partial s}{\partial \odot \mathcal{M}} = - \tfrac{1}{3} \frac{s}{\odot \mathcal{M}} \qquad \frac{\partial p}{\partial \odot \mathcal{M}} = 0.$$

Further, if we put

$$Z_i = \frac{a_i'' s_i}{\Delta_i'} \sin 1° \cos B \qquad (i = 1, 2, 3, 4)$$

we find

$$\frac{\partial s}{\partial \rho_1} = - Z_1 \cos v_1 \cos (p - P) \qquad \frac{\partial s}{\partial q_1} = - Z_1 \sin v_1 \cos (p - P)$$

(6)

$$\frac{\partial s}{\partial p_2} = Z_2 \cos v_2 \cos (p - P) \qquad \frac{\partial s}{\partial q_2} = Z_2 \sin v_2 \cos (p - P)$$

$$s \sin 1'' \frac{\partial p}{\partial \rho_1} = Z_1 \cos v_1 \sin (p - P) \qquad s \sin 1'' \frac{\partial p}{\partial q_1} = Z_1 \sin v_1 \sin (p - P)$$

(7)

$$s \sin 1'' \frac{\partial p}{\partial \rho_2} = - Z_2 \cos v_2 \sin (p - P) \qquad s \sin 1'' \frac{\partial p}{\partial q_2} = - Z_2 \sin v_2 \sin (p - P)$$

If we denote by $\Delta\rho_1$, $\Delta\rho_2$, Δv_1 etc corrections to the adopted radii vectores, longitudes etc, and if we put $\mu = \frac{100}{3} \frac{\Delta \odot \mathcal{M}}{\odot \mathcal{M}}$

and $z = \frac{\varkappa^2}{200}$ = the correction for 100" due to „crossing through" (supposed different for the two observers), the equations of condition become in distance

(Ia)

$$O - C = - \frac{s}{100} \mu - \frac{100}{s} z + \rho_1 \frac{\partial s}{\partial \rho_1} \frac{\Delta \rho_1}{\rho_1} + \rho_2 \frac{\partial s}{\partial s_2} \frac{\Delta \rho_2}{\rho_2} + \frac{\partial s}{\partial v_1} \Delta v_1 + \frac{\partial s}{\partial v_2} \Delta v_2 +$$

$$+ \frac{\partial s}{\partial \rho_1} \Delta \rho_1 + \frac{\partial s}{\partial q_1} \Delta q_1 + \frac{\partial s}{\partial \rho_2} \Delta \rho_2 + \frac{\partial s}{\partial q_2} \Delta q_2$$

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and in position-angle

$$s \sin 1'' \,(O-C) = s \sin 1'' \Big\{ f_1 \frac{\partial p}{\partial \varepsilon_1} \frac{\Delta \varepsilon_1}{\rho_1} + f_2 \frac{\partial p}{\partial \varepsilon_2} \frac{\Delta \varepsilon_2}{\rho_2} + \frac{\partial p}{\partial v_1} \Delta v + \frac{\partial p}{\partial v_2} \Delta v_2 + \frac{\partial p}{\partial p_1} \Delta p_1 + \frac{\partial p}{\partial q_1} \Delta q_1 + \frac{\partial p}{\partial p_2} \Delta p_2 + \frac{\partial p}{\partial q_2} \Delta q_2 \Big\} \tag{Ib}$$

The corrections Δv and Δp must now be expressed in terms of the corrections to the fundamental constants of the theory. The question thus presents itself, which constants must be selected for correction.

The fundamental data on which a self-consistent theory like Souillart's rests are

ΔM = reciprocal of the mass of the whole system,
J = coefficient depending on the compression of Jupiter,
i and Ω = inclination and ascending node
e and ϖ = excentricity and perijove
n = mean daily motion
ε = longitude at epoch
m = mass
} of each of the satellites

The mean motions n must be assumed to be known, as already mentioned. ΔM appears already explicitly in the equations.

i and Ω are represented by p and q, which have been introduced because the coefficients for these quantities are more easily computed, and are independent of the adopted values of i and Ω.

For ε we have $\frac{\partial s}{\partial \varepsilon_i} = \frac{\partial s}{\partial v_i}$ and $\frac{\partial p}{\partial \varepsilon} = \frac{\partial p}{\partial v_i}$; these quantities require no additional computation.

Between the longitudes and mean motions of the three inner satellites there exist the relations

$$n_1 - 3n_2 + 2n_3 = 0$$
$$\vartheta = l_1 - 3l_2 + 2l_3 = 180°$$

These relations are rigorous if the *mean* values of the involved quantities are considered. Laplace found theoretically that the deviations of the actual values at a certain epoch from the mean values can be expressed as a periodic inequality, which he called libration, and which is distributed in a fixed proportion among the three satellites. Delambre was unable to find any trace of this libration from the observations. The probability is thus that it will be sufficiently exact to consider the above equations as rigorously true for observations extending only over a short interval of time. As however the practical advantage, which would be gained by such a course, *viz* the elimination of one unknown quantity, is not nearly so large as it appears at first sight (as will be explained below), it was thought better provisionally not to introduce the condition $\Delta\varepsilon_1 - 3\,\Delta\varepsilon_2 + 2\,\Delta\varepsilon_3 = 0$.

The quantity J can not be determined with any weight from these observations, and must therefore be taken as correct. The only means of determining it would be from secular perturbations in the nodes or perijovia.

It remains to introduce the unknowns e, ϖ and m for each of the satellites.

These appear in the longitudes and radii vectores as periodic inequalities, analytically represented by expressions of the form:

$$P \sin (\Phi - Q) \text{ for the longitudes,}$$
$$\text{and } P' \cos (\Phi - Q) \text{ for the radii vectores,}$$

where Φ is proportional to the time, Q depends on the perijovia and P and P′ on the excentricities and masses.

Thus to introduce m, e and ϖ into the equations we must first select those periodic inequalities which it is possible to determine with any weight from the observations, that is those of which the periods

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are short in comparison with the period covered by the observations (119 days), and which are at the same time of importance for the determination of the masses and excentricities.

We have then $$\Delta v = \Delta\varepsilon + \Sigma\{\sin\Phi\,\Delta(P\cos Q) - \cos\Phi\,\Delta(P\sin Q)\},$$
$$\Delta\rho = \Sigma\frac{P'}{P}\{\cos\Phi\,\Delta(P\cos Q) + \sin\Phi\,\Delta(P\sin Q)\},$$

where the summation extends over all the selected terms.

I will now enumerate the periodic terms which have actually been taken into account.

1° Each of the satellites has *four* periodic terms of which the period is equal to the period of revolution of the satellite, that is for which $\Phi = l =$ mean longitude. These are of the form:

$$\mathbf{a}_1\sin(l-\varpi_1) \qquad \mathbf{a}_2\sin(l-\varpi_2) \qquad \mathbf{a}_3\sin(l-\varpi_3) \qquad \mathbf{a}_4\sin(l-\varpi_4)$$

for the longitudes; and for the radii vectores:

$$\mathbf{a}'_1\cos(l-\varpi_1) \qquad \mathbf{a}'_2\cos(l-\varpi_2) \qquad \mathbf{a}'_3\cos(l-\varpi_3) \qquad \mathbf{a}'_4\cos(l-\varpi_4)$$

where $\mathbf{a}'_1 = -\frac{1}{2}\sin 1^\circ.\mathbf{a}_1$, $\mathbf{a}'_2 = -\frac{1}{2}\sin 1^\circ.\mathbf{a}_2$ etc., if the longitudes are expressed in degrees, and the radii vectores in the mean distances as unit.

It is not possible to separate these four terms by the observations.

They can be combined in *one* term of the form:

$2E\sin(l-\Omega)$ in longitude,

and $-E\sin 1^\circ\cos(l-\Omega)$ in radius vector,

where
$$2E\sin\Omega = \mathbf{a}_1\sin\varpi_1 + \mathbf{a}_2\sin\varpi_2 + \mathbf{a}_3\sin\varpi_3 + \mathbf{a}_4\sin\varpi_4$$
$$2E\cos\Omega = \mathbf{a}_1\cos\varpi_1 + \mathbf{a}_2\cos\varpi_2 + \mathbf{a}_3\cos\varpi_3 + \mathbf{a}_4\cos\varpi_4$$

The $\mathbf{a}$'s, and therefore E and Ω involve not only the excentricities but also the masses of the satellites.

If we put $h = 2E\sin\Omega$, $k = 2E\cos\Omega$, the periodic terms become in longitude

$$k\sin l - h\cos l$$

and in radius vector $$-\tfrac{1}{2}\sin 1^\circ(k\cos l + h\sin l).$$

These terms include the inequalities depending on Marth's arguments γ_1, δ_1; $\gamma_2, \delta_2, \varepsilon_2$; $\alpha_3, \beta_3, \iota_3$; α_4, γ_4. Their periods are respectively $1^d.77$, $3^d.55$, $7^d.15$ and $16^d.69$.

2° The principal inequalities of the three inner satellites are very large owing to the circumstance that the mean motions are very approximately proportional to small whole numbers. Their expressions are

Satellite	*Marth's argument*	*in longitude*	*in radius vector*	*period*	*max. geoc. effect.*
I	α_1	$0''.432\sin 2(l_1-l_2)$	$-0.00378\cos 2(l_1-l_2)$	$1^d.76$	$1''.08$
II	α_2	$-1.203\sin(l_1-l_2)$	$+0.00901\cos(l_1-l_2)$	3.52	3.66
III	γ_3	$-0.063\sin(l_2-l_3)$	$+0.00058\cos(l_2-l_3)$	7.05	0.37.

In the last column is given the maximum effect of the perturbation on the place of the satellite as seen from the earth, when Jupiter is in opposition to the sun ($\log\Delta = 0.6$).

The coefficients in radius vector are again very approximately equal to those in longitude multiplied by $-\frac{1}{2}\sin 1''$. These coefficients involve the masses m_1, m_2 and m_3 only.

If we call the corrections to the coefficients in longitude x_1, x_2, x_3, the differential coefficient of x_1 is in longitude $\sin\alpha_1$ and in radius vector $-\frac{1}{2}\sin 1''\cos\alpha_1$ and similarly for x_2 and x_3. The angles α_1, α_2, γ_3 can be taken from the computations of longitudes and radii vectores for the time of each observation

3°. In addition to these large inequalities I have taken into account such terms of short period, of which the maximum effect on the position of the satellite as seen from the earth is at least equal to $0''.05$. These are the following.

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Satellite.	*Marth's argument.*	*in longitude.*	*in radius vector.*	*period.*	*max. geoc. effect.* *longit.*	*radius vect.*
II	β_2	$-0°.013 \sin (l_2-l_3)$	$+0.00007 \cos (l_2-l_3)$	d 7.05	0".05	0".01
		$+0.006 \sin 3 (l_2-l_3)$	$-0.00006 \cos 3 (l_2-l_3)$		0.02	0.01
III	δ_3	$-0.004 \sin (l_3-l_4)$	$+0.00002 \cos (l_3-l_4)$	12.53	0.02	0.01
		$-0.014 \sin 2 (l_3-l_4)$	$+0.00013 \cos 2 (l_3-l_4)$		0 07	0.05
IV	β_4	$-0.003 \sin (l_3-l_4)$	$+0\ 00011 \cos (l_3-l_4)$	12.53	0.02	0.07
		$+0.001 \sin 2 (l_3-l_4)$	$-0.00002 \cos 2 (l_3-l_4)$		0.01	0 01
	δ_4	$+0.006 \sin (l_4-2l_\odot+\varpi_4)$	$-0.00005 \cos (l_4-2l_\odot+\varpi_4)$	16.69	0.06	0.03

The last of these terms involves only the mass of the sun, and l_4 and ϖ_4. For the determination of these last quantities it is of no value compared with the term depending on α_4 (see under 1°) of which the maximum geocentric effect is 6".95. It has therefore been omitted.

The coefficients of all the inequalities depending on multiples of the same argument are proportional to the same mass, *viz.* in satellite II to m_3, in III to m_4 and in IV to m_3.

According to the above mentioned condition only one term for each of the satellites II, III and IV should be taken into account. The other terms are however of so little importance that it makes practically no difference in the differential coefficients, whether we take as the quantity to be introduced into the equations the whole of the inequality depending on multiples of the same argument, or only its most important term. I have therefore, in drawing up the form of the differential coefficients to be computed, been guided entirely by practical considerations regarding the shortness and easiness of computation.

If we call $\mathbf{D}_3$ the sum of the terms in the longitude of satellite III depending on l_3-l_4 and its multiples, that is on Marth's argument δ_3; and $\mathbf{D}'$ the sum of the corresponding terms in the radius vector, then $\mathbf{D}_3$ and $\mathbf{D}'_3 + 0.000144$ can be taken at once from the computations of longitude and radius vector for each epoch of observation, or can be interpolated from Marth's tables. $\mathbf{D}_3$ and $\mathbf{D}'_3$ being both proportional to m_4 we have

$$\frac{\Delta \mathbf{D}_3}{\mathbf{D}_3} = \frac{\Delta m_4}{m_4} = \frac{\Delta \mathbf{D}'_3}{\mathbf{D}'_3} = 10\, y_3 .$$

and if y_3 is introduced as an unknown the differential coefficients in longitude and radius vector become simply $10\,\mathbf{D}_3$ and $10\,\mathbf{D}'_3$.

Similarly if $\mathbf{B}_4$ and $\mathbf{B}'_4$ denote the sums of the terms in the longitude and radius vector of satellite IV, depending on multiples of l_3-l_4 (Marth's β_4), the quantity

$$\frac{\Delta (\mathbf{B}_4)}{10\ \mathbf{B}_4} = \frac{\Delta (\mathbf{B}'_4)}{10\,\mathbf{B}'_4} = y_4$$

has been introduced as an unknown.

A few words must be said about the introduction of the factor 10. If it had been my object to make these differential coefficients of approximately the same order of magnitude as those of the h's, k's and x's, it would have been better to introduce a factor 100 instead of 10; the accuracy however with which the quantities $\mathbf{D}_3$, $\mathbf{D}'_3$, $\mathbf{B}_4$, $\mathbf{B}'_4$ are computed is not sufficient to allow multiplication by 100, and also by introducing the factor 10 the magnitude of the coefficients is in better agreement with the relative importance of the different unknowns for a determination of the masses.

In the case of satellite II, only the first of the terms depending on l_2-l_3 has been taken into account. If $\frac{1}{10}\, y_2$ represents the correction to the coefficient in longitude, the differential coefficients become

in longitude $\quad 0.1 \sin \beta_2$

in radius vector $\quad -0.0273 \sin 1° \cos \beta_2$

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By the introduction of the factor $\frac{1}{10}$ the coefficient of y_2 becomes of about the same order of magnitude as those of y_3 and y_4, which are of the same relative importance for the determination of the masses. It would have been better perhaps for the sake of uniformity to adopt the same method of computation for y_2 as for y_3 and y_4, but the matter is of no great importance, and by rejecting the two small terms depending on multiples of β_2 the coefficients become independent of the error in Marth's tables, which was mentioned in the beginning of this chapter.

We have thus finally:

$$\begin{aligned}
\Delta v_1 &= \Delta\varepsilon_1 - \cos l_1 \Delta h_1 + \sin l_1 \Delta k_1 + \sin \alpha_1 x_1 \\
\Delta \rho_1 &= -\tfrac{1}{2} \sin 1'' \sin l_1 \Delta h_1 - \tfrac{1}{2} \sin 1'' \cos l_1 \Delta k_1 - \tfrac{1}{2} \sin 1'' \cos \alpha_1 x_1 \\
\Delta v_2 &= \Delta\varepsilon_2 - \cos l_2 \Delta h_2 + \sin l_2 \Delta k_2 + \sin \alpha_2 x_2 + 0.1 \sin \beta_2 y_2 \\
\Delta \rho_2 &= -\tfrac{1}{2} \sin 1'' \sin l_2 \Delta h_2 - \tfrac{1}{2} \sin 1'' \cos l_2 \Delta k_2 - \tfrac{1}{2} \sin 1'' \cos \alpha_2 x_2 - 0.0273 \cos \beta_2 y_2 \\
\text{(II)} \quad \Delta v_3 &= \Delta\varepsilon_3 - \cos l_3 \Delta h_3 + \sin l_3 \Delta k_3 + \sin \gamma_3 x_3 + 10\, \mathbf{D}_3 y_3 \\
\Delta \rho_3 &= -\tfrac{1}{2} \sin 1'' \sin l_3 \Delta h_3 - \tfrac{1}{2} \sin 1'' \cos l_3 \Delta k_3 - \sin 1'' \cos \gamma_3 x_3 + 10\, \mathbf{D}'_3 y_3 \\
\Delta v_4 &= \Delta\varepsilon_4 - \cos l_4 \Delta h_4 + \sin l_4 \Delta k_4 + 10\, \mathbf{B}_4 y_4 \\
\Delta \rho_4 &= -\tfrac{1}{2} \sin 1'' \sin l_4 \Delta h_4 - \tfrac{1}{2} \sin 1'' \cos l_4 \Delta k_4 + 10\, \mathbf{B}'_4 y_4
\end{aligned}$$

The angles α_1, α_2, β_2, γ_3 are Marth's arguments and represent respectively $2(l_1 - l_2)$, $l_1 - l_3 + 180°$, $l_2 - l_3 + 180°$, $l_2 - l_3 + 180°$.

These values have been introduced into the equations (I) and so the complete equations of condition are formed. In this part of the work 4-figure logarithms were used. All the computations have been done in duplicate by Mr S. S. Hough, chief-assistent at the Cape Observatory, and the equations were finally written out from my own computations and then read over with the duplicate. The coefficients in the final equations were rounded off to two decimal places. The complete equations are given in the tables at the end of this chapter.

The next step would now be to express the h's, k's, x's and y's in terms of the e's, ϖ's and m's. The approximate theoretical expressions of the x's and y's are

$$\begin{aligned}
\tfrac{1}{1000}\, x_1 &= 20\, \Delta m_2 & \qquad \tfrac{1}{1000}\, y_2 &= 1.7\, \Delta m_3 \\
\tfrac{1}{1000}\, x_2 &= 11\, \Delta m_3 + 6\, \Delta m_1 & \qquad \tfrac{1}{1000}\, y_2 &= 2.4\, \Delta m_9 \\
\tfrac{1}{1000}\, x_3 &= 3\, \Delta m_2 & \qquad \tfrac{1}{1000}\, y_4 &= 1.1\, \Delta m_3
\end{aligned}$$

The coefficients of the x's are about ten times as large as those of the y's, it appears therefore that only two combinations of the masses *viz.* Δm_2 and $11\, \Delta m_3 + 6\, \Delta m_1$ can be determined with such weight that the resulting values can be expected to be real corrections to the adopted masses.

The x's contain in addition to the terms of the first order in the masses considerable terms of the second order, which amount to respectively $\frac{1}{18}$, $\frac{1}{21}$ and $\frac{1}{7}$ of the terms of the first order.

The y's contain no terms of higher orders than the first. The factors of the masses in these terms are pure functions of the ratios a_1/a_2 etc. between the mean distances, and therefore to be regarded as known quantities. It is thus allowable to introduce one unknown instead of y_2 and y_4 which are proportional to the same mass. I find from Souillart's data

$$y_2/y_4 = 1.468305$$

The h's and k's, as already pointed out, involve excentricities, perijovia *and masses*, and cannot be neglected even in a first approximation to a determination of the masses. We have thus for the determination of corrections to the 12 elements e, ϖ, m only 10 quantities, which are determinable with considerable weight from the observations.

The masses of the satellites can therefore not be determined from this series of observations without

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combination with other observations made at such a distance of time as to allow a good determination of secular perturbations to be made. It appears thus useless to derive from these observations values of the masses, which cannot be final, but the result of this investigation must be presented in the form most suitable for comparison with older or more recent observations. I believe that this condition is best fulfilled by a derivation of corrections to the coefficients of the more important periodic inequalities for the mean epoch of the observations. Accordingly the masses, excentricities and perijovia have not been introduced into the equations of condition.

We have thus finally the following unknown quantities.

z_G and z_F	2
μ	1
$\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_3$, $\Delta\varepsilon_4$	4
Δp_1, Δq_1, Δp_2, Δq_2, Δp_3, Δq_3, Δp_4, Δq_4	8
Δh_1, Δk_1, Δh_2, Δk_2, Δh_3, Δk_3, Δh_4, Δk_4	8
x_1, x_2, x_3	3
y_2, y_3, y_4	3
total number of unknowns	29

The conditions $\Delta\varepsilon_1 = 3\,\Delta\varepsilon_2 - 2\,\Delta\varepsilon_3$ and $y_2 = 1.468\,y_4$ have not been introduced into the equations of condition. The introduction of these conditions, while reducing the *total* number of unknowns by two, would not diminish the number of unknowns in the individual equations, as will be seen from the following little table, where in the column A are given the numbers of unknowns in the equations derived from the distance-observations of the different pairs of satellites as actually used, and in column B these same numbers as they would be if $\Delta\varepsilon_1$ and y_2 were eliminated. The number of unknowns in the position-angle equations is always two less than in the corresponding distance equations.

Pair	A	B	B—A
I—II	15	15	0
I—III	15	15	0
I—IV	14	15	+ 1
II—III	16	16	0
II—IV	15	14	— 1
III—IV	15	15	0

It appears that the elimination of $\Delta\varepsilon_1$ and y_2 does not reduce the number of multiplications needed for the formation of the normal equations, whilst it would complicate the computation of the coefficients and involve more liability to mistakes than an eventual subsequent alteration of the normal equations.

The weights of the individual equations must to a large extent depend on judgment. The observations are too few and the number of unknowns is too large to allow a determination to be made, from the residuals of a first approximate solution, of the relative weights of observations by the two observers, made in different definition, of different distances and by different methods of observing. In assigning weights I have been guided by the following principles.

Observations by the two observers have the same weight. This is in accordance with previous experience in the Victoria-triangulation.

The position-angles (after reduction to the great circle) are entitled to the same weight as the distances. This was also found to be the case in the Victoria-triangulation work.

Distances measured with the images at half or double distance have received half weight. There

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can be no doubt that this method is less accurate than the method of crossing through used for long distances. All position-angle equations have the same weight, whether the images were placed at half or double distance or the measures were made in the ordinary way.

Distances measured by crossing through have received weight unity independent of the distance. It is very probable that short distances, say under 400" or 500", where the curvature of the path described by the images in crossing is noticeable, are less accurately measured than distances of some thousands of seconds, but in the absense of any definite experience about the measuring of such short distances it has been thought better to give a uniform weight to all distances observed by crossing through, whether ψ was recorded or not.

Double weight was given to observations consisting of 8 pointings instead of 4. Such observations are distinguished in the tables by the addition of an asterisk (*) before the observed value.

Half weight was given 1° to all observations made with estimated quality of images 3,

2° to observations 50 and 499 in both of which the last pointing was not printed, on account of the consequent asymmetry,

3° to two observations (N^rs 328—9 and 497—8) on account of their unusually difficult nature.

A weight ¼ was given to all observations made when the quality of images was 3-4.

Entirely rejected were: 1° all observations on Nov. 19 and Nov. 30. On both dates the definition was very bad and there is no chronometer comparison.

2° The observations 142 and 379 in which one pointing is not printed, while it is uncertain which pointing, so that the epoch becomes uncertain.

3° All observations in which the original readings were altered (*e. g.* by 1° in position-angle, or one scale-division in distance), *viz.* n^rs 131, 235, 441, 477. Under this head come also the obs^ns 87 and 283 which were rejected on account of a discordance of 30′ in position-angle, though this discordance could not be explained by an evident mistake in the recorded readings.

4° The observation 459—60 where a distance of 6" was measured by „crossing".

In order to facilitate the multiplication of the equations by the square root of the weights, 0.49 was substituted for ½

Finally I give a short *résumé* of the constants adopted for the computation of the tabular places and of the periodic inequalities which are discussed in this chapter.

I The mean epoch of the observations is:

1891 *October* 2, *Greenwich mean noon* = 1891.75.

The value of the constants given below are all for this epoch.

II The position of the adopted fundamental plane, and of the point 0 from which the longitudes in that plane are reckoned are:

relatively to the ecliptic and mean equinox of date:

ascending node . . . $\mathcal{N}$ = 336° 15′ 6″.54
inclination . . . . . $\mathcal{J}$ = 2 9 1.09
longitude of 0 . . . $\mathcal{O}$ = 135 20 31.31
daily motion of 0 . . . . . 0.12109

relatively to the true equator and true equinox of date:

ascending node . . . N = −2″.0164
inclination . . . . . I = 25.4369
longitude of 0 . . . O = 135.1575.

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From these quantities we derive, or get by interpolation from Marth's tables:

$$L = 204^\circ.414$$
$$B = +\ 1.594$$
$$P = 335.613 = 335^\circ 36' 47''.$$

III The elements and constants adopted by Marth, and used in the computation of his tables are:

$$\mu = 1047.700$$

$n_1 = 203^\circ.4889560$	$\log a_1 = 7.450\,150 - 10$	$\varepsilon_1 = 178^\circ.2725$	$\varpi_1 = 235^\circ.6034$
$n_2 = 101.3747256$	$\log a_2 = 7.651\,788 - 10$	$\varepsilon_2 = 263.7747$	$\varpi_2 = 45.5259$
$n_3 = 50.3176104$	$\log a_3 = 7.854\,580 - 10$	$\varepsilon_3 = 35.9258$	$\varpi_3 = 205.4592$
$n_4 = 21.5710740$	$\log a_4 = 8.099\,795 - 10$	$\varepsilon_4 = 286.8757$	$\varpi_4 = 146.7810$

$i_1 = 0^\circ.0081$	$\Omega_1 = 250^\circ.96$	$p_1 = +0^\circ.0076$	$q_1 = -0^\circ.0026$
$i_2 = 0.4915$	$\Omega_2 = 237.87$	$p_2 = +0.4162$	$q_2 = -0.2614$
$i_3 = 0.1358$	$\Omega_3 = 239.13$	$p_3 = +0.1166$	$q_3 = -0.0697$
$i_4 = 0.3470$	$\Omega_4 = 330.77$	$p_4 = +0.1694$	$q_4 = -0.3028$

The values of p and q have been computed from the adopted values of i and Ω. The following values of h and k have been computed from the periodic inequalities adopted by Marth.

$h_1 = -0^\circ.0030$	$k_1 = -0^\circ.0034$
$h_2 = +0.0059$	$k_2 = -0.0211$
$h_3 = -0.0194$	$k_3 = -0.1727$
$h_4 = -0.4629$	$k_4 = -0.6813$

IV The coefficients of the inequalities are taken from Souillart, and rest on the following adopted masses and value of the constant depending on the compression of Jupiter:

$$m_1 = 0.0000\,16877$$
$$m_2 = 0.0000\,23227 \qquad Jb^2 = 0.0247187 \ (b = 1)$$
$$m_3 = 0.0000\,88437$$
$$m_4 = 0.0000\,42475$$

Table III shews the periodic terms discussed in this chapter, together with the corresponding unknowns and their approximate theoretical expressions.

The second column gives the arguments, the third column gives the coefficients of the sines of these arguments in the expressions for the longitudes, the fourth column gives the coefficients of the cosines in the expressions for the radii vectores. Thus *e.g.*

$v_1 = \varepsilon_1 + n_1 t + 1554''.759 \sin 2(l_1 - l_2) + 8''.796 \sin (l_1 - \varpi_1) + 8''.145 \sin (l_1 - \varpi_3) +$ (terms of which the corrections are not introduced as unknowns)

$r_1 = 1 - 0.0037751 \cos 2(l_1 - l_2) - 0.0000\,213 \cos (l_1 - \varpi_1) - 0.0000196 \cos (l_1 - \varpi_3) -$ (terms of which . . . . .)

The other columns are amply explained by their headings.

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TABLE III. — PERIODIC TERMS AND UNKNOWNS.

Satellite	Arguments	Longitude. Coefficient of sine	Radius Vector. Coefficient of cosine	Marth's argument	Period	Maximum geocentric effect	Corresponding unknown	Approximate theoretical expression
		"			d	"		
I	$2(l_1-l_2)$	+ 1554.759	−0.0037751	α_1	1.76	1.08	x_1	19893 Δm_1 + second order terms
	$l_1-\varpi_1$	+ 8.796	−0.0000213	γ_1	1.77	0.005	$h_1\ k_1$	involving e's, ϖ's and m's
	$l_1-\varpi_3$	+ 8.145	−0.0000196	δ_1				
II	l_1-l_2	− 3692.981	+ 0.0090095	$\alpha_2+180°$	3.52	3.66	x_2	10925 Δm_3 + 6280 Δm_1 + second order terms
	$l_2-\varpi_2$	+ 45.246	−0.0001096	γ_2	3.55	0.08	$h_2\ k_2$	involving e's, ϖ's and m's
	$l_2-\varpi_3$	+ 79.680	−0.0001931	δ_2				
	$l_2-\varpi_4$	+ 42.593	−0.0001032	ε_2				
	l_2-l_3	− 48.578	+ 0.0000672	β_2+180	7.05	0.05	y_2	1660 Δm_3
III	l_2-l_3	− 228.473	+ 0.0005827	γ_3+180	7.05	0.37	x_3	3117 Δm_2 second orderterms
	$l_3-\varpi_2$	− 1.936	+ 0.0000050	ι_3+180	7.15	0.76	$h_3\ k_3$	involving e's, ϖ's and m's, especially m_4
	$l_3-\varpi_3$	+ 462.880	−0.0011219	α_3				
	$l_3-\varpi_4$	+ 241.905	−0.0005864	β_3				
	l_3-l_4	− 14.620	+ 0.0000221	δ_3+180	12.53	0.09	y_3	2354 Δm_4
	$2(l_3-l_4)$	+ 49.710	−0.0001320	$2\delta_3$				
	$3(l_3-l_4)$	+ 3.503	−0.0000117	$3\delta_3+180$				
	$4(l_3-l_4)$	+ 0.814	−0.0000031	$4\delta_3$				
	$5(l_3-l_4)$	+ 0.258	−0.0000011	$5\delta_3+180$				
IV	$l_4-\varpi_3$	− 59.063	+ 0.0001431	γ_4+180	16.69	6.95	$h_4\ k_4$	involving e's, ϖ's and m's
	$l_4-\varpi_4$	+ 2995.688	−0.0072597	α_4				
	l_3-l_4	− 9.189	+ 0.0001105	β_4+180	12.53	0.07	y_4	1131 Δm_3
	$2(l_3-l_4)$	− 4.571	+ 0.0000201	$2\beta_4$				
	$3(l_3-l_4)$	− 0.951	+ 0.0000047	$3\beta_4+180$				

In the following tables are given the epochs of observation, the observed and computed values, and the equations of condition. The meaning of the different columns is sufficiently explained by their headings. A few particulars, which have been explained in this and the preceding chapter, may be repeated here.

Greenwich time. This is the time of observation, corrected for aberration using the distance of Jupiter from the earth. (see page 7).

Computed distance and *computed position-angle.* These are for the epochs of the preceding column.

Observed distance and *observed position-angle.* An asterisk (*) in these columns denotes that eight pointings were taken.

Method (in the tables of distances) c denotes the ordinary method of crossing.
ψ „ that the amount of crossing was measured.
d „ „ the images were at double distance.
h „ „ „ „ „ „ half „

O—C. This does *not* include the correction for differential aberration (see page 18).

Differential Aberration. This is the correction to be added to the O—C of the preceding column.

Residuals. The residuals belonging to rejected observations are included in brackets.

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TABLE IV. SATELLITES I and

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	OBSERV… Coeffic… μ	z_G	x_1	Δh_1	Δk_1
	1891			h m	"	"							
3	Aug. 17	2	2	9 25.13	109.749	109.567	c	1.0	— 1.10	— 0.91	+ 0.89	— 1.27	+ 1.48
20	19	2-3	3	9 32.42	311.990	311.809	c	1.0	— 3.12	— 0.32	— 0.09	+ 1.16	— 0.54
34	20	2	3	9 1.93	316.322	316.377	c	1.0	— 3.16	— 0.32	— 0.24	— 1.55	+ 0.30
44	21	3	3-4	7 59.36	4.647	*4.601	d	0.5	— 0.05	0.00	— 0.24	— 1.70	+ 0.34
50		2	3-4	8 58.46	4.605	*4.577	d	0.5	— 0.05	0.00	— 0.22	— 1.58	+ 0.42
64	24	2-3	2-3	9 12.00	154.277	153.989	c	1.0	— 1.54	— 0.65	+ 1.03	— 1.65	+ 1.58
80	25	2-3	2-3	8 50.76	39.190	39.196	η'	1.0	— 0.39	0.00	+ 0.98	— 1.24	+ 1.38
81		2-3	2-3	9 1.24	38.696	38.736	η'	1.0	— 0.39	0.00	+ 0.96	— 1.21	+ 1.36
103	29	2-3	2-3	8 49.95	128.656	128.904	c	1.0	— 1.29	— 0.78	+ 0.01	+ 2.18	— 0.50
121	31	2-3	2-3	8 58.74	197.328	197.171	c	1.0	— 1.97	— 0.51	+ 1.01	— 2.03	+ 1.46
142	Sept. 1	—		8 55.12	49.848	50.043	η'	0.0	— 0.50	0.00	+ 1.19	— 1.54	+ 1.51
145	5	3	3-4	6 57.44	39.795	39.600	η'	0.5	— 0.40	0.00	— 0.01	+ 1.56	— 0.24
164	7	1-2	2-3	7 11.26	261.974	261.755	c	1.0	— 2.62	— 0.38	+ 0.56	— 2.35	+ 0.84
194	10	2	2-3	7 24.85	196.042	196.354	c	1.0	— 1.96	— 0.51	+ 1.14	— 1.21	+ 1.24
204	13	2	3	6 59.91	268.013	267.782	c	1.0	— 2.68	— 0.37	— 0.16	+ 1.77	— 0.23
221	14	3	3 & 3-4	6 38.73	286.619	286.448	c	0.5	— 2.87	— 0.35	+ 0.36	— 2.19	+ 0.42
249	19	2	3	5 51.76	48.702	48.875	c	1.0	— 0.49	— 2.05	+ 0.86	— 1.08	+ 0.82
269	20	1-2	2	6 8.86	324.779	324.750	c	1.0	— 3.25	— 0.31	— 0.41	+ 1.31	— 0.34
294	21	1	1-2	7 3.67	289.421	289.417	c	1.0	— 2.89	— 0.35	+ 0.36	— 1.98	+ 0.24
300	22	2	2-3	6 13.22	44.039	44.294	η'	1.0	— 0.44	0.00	+ 0.64	— 2.23	+ 0.47
329	24	2-3	—	5 43.91	10.059	*10.015	h	0.5	— 0.10	0.00	+ 0.30	— 0.48	+ 0.25
333	28	3	3-4	6 9.00	280.281	280.225	c	0.5	— 2.80	— 0.36	+ 0.42	— 1.48	+ 0.20
335	29	2	3	5 23.12	44.031	44.197	η'	1.0	— 0.44	0.00	+ 0.48	— 1.83	+ 0.18
347	Oct. 4	2-3	2-3	5 44.30	348.583	348.782	c	1.0	— 3.49	— 0.29	— 1.18	+ 1.09	— 0.95
369	9	2-3	2-3	9 11.10	141.298	141.348	c	1.0	— 1.41	— 0.71	+ 1.77	— 1.60	+ 1.36
385	13	1-2	1-2	8 48.84	55.613	55.999	η'	1.0	— 0.56	0.00	+ 0.72	— 1.76	+ 0.13
466	Nov. 5	2-3	3-4	7 22.61	296.021	295.979	c	1.0	— 2.96	— 0.34	— 0.95	+ 1.43	— 0.11
482	6	2	2-3	7 39.38	218.120	217.932	c	1.0	— 2.18	— 0.46	+ 1.34	— 1.92	+ 0.21
498	9	—		7 5.83	7.673	7.141	d	0.5	— 0.08	0.00	+ 1.08	— 0.93	+ 0.64
503	22	2	2-3	6 44.47	57.534	57.194	c	1.0	— 0.58	— 1.75	— 1.65	+ 1.91	— 0.32

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	OBSERV… Coeffic… μ	z_F	x_1	Δh_1	Δk_1
	1891			h m	"	"							
235	Sept. 18	2	2	6 15.15	103.913	103.747	c	0.0	— 1.04	— 0.96	+ 1.30	— 1.23	+ 1.27
400	Oct. 14	2-3	3	8 48.26	108.837	108.601	c	1.0	— 1.09	— 0.92	— 1.12	+ 2.07	— 0.38
424	16	2-3	3	9 13.39	165.818	165.427	c	1.0	— 1.66	— 0.60	+ 1.90	— 1.89	+ 1.28
446	19	2	3	9 6.75	222.665	222.657	c	1.0	— 2.23	— 0.45	+ 0.85	— 1.05	+ 0.42
522	Nov. 30	3	3-4	6 26.99	198.787	198.557	c	0.0	— 1.99	— 0.50	— 1.80	+ 1.90	— 0.40
528	Dec. 8	2	3	6 18.00	155.091	154.647	c	1.0	— 1.55	— 0.65	+ 1.90	— 1.88	+ 0.53

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Δk_2	y_2	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2	O—C	Differential aberration	Residuals	rot. Nr.
								"	"	"	
— 0.52	+ 0.18	— 1.72	— 1.43	— 0.02	— 0.08	— 0.10	+ 0.08	— 0.182	— 0.003	+ 0.037	3
+ 0.63	+ 0.01	— 0.24	— 2.71	— 0.02	+ 0.02	— 0.04	+ 0.01	— 0.181	— 0.005	— 0.179	20
— 2.14	+ 0.06	— 1.09	+ 2.37	+ 0.02	— 0.01	0.00	— 0.04	+ 0.055	+ 0.003	+ 0.111	34
— 0.91	— 0.33	+ 1.48	— 3.26	+ 0.86	— 0.33	— 1.51	0.00	— 0.046	+ 0.004	— 0.177	44
— 1.06	— 0.35	+ 1.75	— 3.43	+ 0.82	— 0.18	— 1.36	— 0.09	— 0.028	+ 0.004	— 0.181	50
— 0.67	+ 0.12	— 2.19	— 0.73	— 0.03	— 0.05	— 0.06	+ 0.07	— 0.288	— 0.006	— 0.036	64
— 2.45	+ 0.38	+ 1.56	— 3.85	+ 0.04	+ 0.23	— 0.26	— 0.28	+ 0.006	+ 0.007	— 0.156	80
— 2.46	+ 0.38	+ 1.51	— 3.83	+ 0.03	+ 0.24	— 0.26	— 0.30	+ 0.040	+ 0.007	— 0.105	81
+ 1.87	+ 0.20	+ 2.12	+ 1.75	— 0.16	+ 0.02	+ 0.01	— 0.25	+ 0.248	+ 0.004	+ 0.103	103
— 0.93	+ 0.07	— 2.48	0.00	— 0.02	— 0.02	— 0.02	+ 0.05	— 0.157	— 0.008	+ 0.101	121
— 2.24	+ 0.40	+ 2.01	— 4.03	+ 0.06	+ 0.13	— 0.18	— 0.14	+ 0.195	+ 0.005	[— 0.008]	142
+ 2.27	+ 0.25	+ 1.11	+ 2.72	— 0.38	+ 0.28	— 0.20	— 0.72	— 0.195	— 0.003	— 0.133	145
— 1.48	— 0.01	— 2.47	+ 1.18	— 0.01	0.00	0.00	+ 0.01	— 0.219	— 0.007	+ 0.001	164
— 2.23	+ 0.10	+ 1.35	+ 3.82	— 0.01	— 0.11	— 0.12	— 0.12	+ 0.312	+ 0.008	+ 0.120	194
+ 0.57	+ 0.03	— 1.44	— 3.07	— 0.05	+ 0.03	— 0.08	+ 0.02	— 0.231	— 0.003	— 0.091	204
— 1.84	— 0.06	— 2.10	+ 1.94	+ 0.01	0.00	0.00	— 0.01	— 0.171	— 0.003	— 0.028	221
— 2.34	— 0.35	+ 0.57	— 3.70	— 0.06	+ 0.25	— 0.27	— 0.31	+ 0.173	+ 0.009	+ 0.123	249
+ 0.35	+ 0.04	— 0.55	— 2.39	— 0.01	+ 0.02	— 0.03	+ 0.02	— 0.029	— 0.004	+ 0.006	269
— 2.00	— 0.09	— 1.77	+ 2.39	+ 0.01	0.00	0.00	— 0.02	— 0.004	0.000	+ 0.083	294
— 0.74	+ 0.35	+ 2.20	— 3.53	+ 0.03	0.00	— 0.04	0.00	+ 0.255	+ 0.002	+ 0.040	300
— 0.28	+ 0.02	+ 0.53	+ 0.91	— 1.78	— 1.46	— 3.47	— 1.06	— 0.044	— 0.002	+ 0.270	329
— 2.17	— 0.12	— 0.94	+ 3.04	+ 0.02	— 0.02	— 0.02	— 0.05	— 0.056	+ 0.005	— 0.073	333
— 0.37	+ 0.30	+ 1.56	— 2.96	+ 0.02	— 0.01	— 0.03	+ 0.01	+ 0.166	+ 0.003	+ 0.012	335
+ 0.43	+ 0.06	+ 0.87	— 1.05	0.00	— 0.01	+ 0.01	— 0.01	+ 0.199	0.000	+ 0.099	347
— 0.79	— 0.06	— 1.97	+ 0.07	— 0.01	— 0.02	— 0.02	+ 0.04	+ 0.050	— 0.005	+ 0.208	369
— 0.22	+ 0.27	+ 1.49	— 2.63	0.00	0.00	+ 0.01	0.00	+ 0.386	+ 0.002	+ 0.230	385
+ 0.17	— 0.08	— 1.02	— 1.71	— 0.01	+ 0.01	— 0.02	+ 0.02	— 0.042	0.000	+ 0.056	466
— 1.86	+ 0.16	— 1.80	+ 2.67	+ 0.02	0.00	— 0.01	— 0.02	— 0.188	0.000	— 0.112	482
— 0.44	— 0.09	+ 1.08	+ 1.93	— 1.15	— 1.44	— 2.89	— 0.32	— 0.532	+ 0.002	— 0.430	498
+ 1.88	+ 0.20	+ 1.85	+ 2.68	— 0.24	+ 0.03	— 0.19	— 0.34	— 0.340	— 0.001	— 0.253	503

NLAY

Δk_2	y_2	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2	O—C	Differential aberration	Residuals	rot. Nr.
								"	"	"	
— 0.45	— 0.14	— 1.42	— 1.08	— 0.01	— 0.06	— 0.06	+ 0.07	— 0.166	— 0.002	[+ 0.219]	235
+ 1.92	— 0.22	+ 2.13	+ 2.32	— 0.16	+ 0.02	— 0.06	— 0.25	— 0.236	+ 0.002	— 0.082	400
— 1.08	+ 0.02	— 2.24	+ 0.72	— 0.01	— 0.01	0.00	+ 0.02	— 0.391	— 0.006	— 0.062	424
— 1.94	— 0.17	— 0.08	+ 3.48	+ 0.02	— 0.05	— 0.06	— 0.06	— 0.008	+ 0.008	+ 0.016	446
+ 0.18	+ 0.12	— 1.90	— 2.04	— 0.04	0.00	— 0.06	+ 0.03	— 0.230	+ 0.003	[+ 0.271]	522
— 1.79	— 0.20	— 1.92	+ 2.56	+ 0.02	0.00	— 0.01	— 0.02	— 0.444	— 0.001	— 0.229	528

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TABLE V. SATELLITES I and II

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficients Δp_1	Δq_1	Δp_2	Δq_2	x_1	Δ
	1891			h m	° ′ ″	° ′ ″							
4	Aug. 17	2	2	9 34.60	66 59 5	66 57 10	1.0	— 0.50	— 2.48	— 3.15	+ 2.51	+ 0.03	+
19	19	2-3	2-3	9 30.65	235 39 11	245 35 8	1.0	— 1.46	+ 2.06	— 3.81	+ 1.19	— 0.08	−
33	20	2	3	9 0.70	64 29 58	64 32 12	1.0	— 2.08	+ 1.45	+ 0.46	+ 3.98	+ 0.06	+
43	21	3	3-4	7 59.36	86 25 11	*84 22 42	1.0	+ 2.20	— 0.84	— 3.79	+ 0.01	+ 0.16	+
49		2	3-4	8 58.65	84 34 17	*81 41 57	2.0	+ 2.33	— 0.51	— 3.83	— 0.26	+ 0.13	+
63	24	2-3	2-3	9 12.51	66 19 18	66 19 37	1.0	— 1.28	— 2.20	— 2.59	+ 3.13	+ 0.03	+
79	25	2-3	2-3	8 52.00	70 29 50	69 47 2	1.0	+ 0.39	+ 2.50	— 2.72	— 3.02	— 0.03	+
102	29	2-3	2-3	8 50.90	241 39 30	241 51 14	1.0	+ 2.54	— 0.24	— 0.21	+ 4.05	— 0.04	+
120	31	2-3	2-3	8 58.11	65 55 32	66 0 19	1.0	— 1.89	— 1.73	— 1.93	+ 3.58	+ 0.03	+
141	Sept. 1	—		8 55.60	68 26 45	67 48 43	1.0	+ 1.03	+ 2.33	— 3.23	— 2.52	— 0.01	+
144	5	3	3-4	6 57.30	234 42 10	235 19 27	0.5	+ 2.02	— 1.50	+ 1.06	+ 3.87	— 0.07	+
163	7	1-2	2-3	7 12.32	65 28 10	65 30 53	1.0	— 2.51	— 0.53	— 0.74	+ 4.00	+ 0.03	+
193	10	2	2-3	7 24.38	62 51 43	62 59 0	1.0	+ 0.23	+ 2.54	+ 2.84	+ 2.86	+ 0.12	−
203	13	2	3	7 0.61	246 35 26	246 29 40	1.0	— 2.22	+ 1.23	— 3.90	+ 0.93	— 0.05	−
220	14	3	3 & 3-4	6 36.84	65 13 28	65 16 19	0.5	— 2.53	+ 0.36	+ 0.16	+ 4.04	+ 0.04	+
248	19	2	3	5 51.62	71 15 31	70 36 30	1.0	— 0.57	+ 2.46	— 2.62	— 3.07	— 0.02	+
268	20	1-2	2	6 8.38	245 57 55	245 57 15	1.0	— 1.56	+ 1.99	— 3.54	+ 1.85	— 0.06	−
293	21	1	1-2	7 3.40	65 7 35	65 11 43	1.0	— 2.38	+ 0.87	+ 0.75	+ 3.95	+ 0.04	+
299	22	2	2-3	6 12.42	66 4 45	65 38 34	1.0	+ 2.51	— 0.17	— 4.05	+ 0.29	+ 0.02	+
328	24	2-3	—	5 43.91	349 4 41	*353 54 21	1.0	+ 0.43	+ 0.35	+ 0.85	+ 0.26	+ 1.38	−
332	28	3	3-4	6 8.75	64 49 13	64 51 59	0.5	— 1.83	+ 1.72	+ 1.68	+ 3.60	+ 0.05	+
334	29	2	3	5 23.66	66 3 19	65 52 35	1.0	+ 2.23	— 1.12	— 3.82	+ 1.23	+ 0.03	+
346	Oct. 4	2-3	2-3	5 44.34	245 23 2	245 21 41	1.0	— 0.15	+ 2.47	— 2.50	+ 3.00	— 0.06	−
368	9	2-3	2-3	9 10.97	66 15 6	66 17 5	1.0	— 1.22	— 2.13	— 1.51	+ 3.62	+ 0.03	+
384	13	1-2	1-2	8 48.84	65 36 32	65 38 4	1.0	+ 2.12	— 1.16	— 3.56	+ 1.60	+ 0.03	+
465	Nov. 5	2-3	3-4	7 22.44	246 14 49	246 14 48	1.0	— 1.72	+ 1.47	— 2.79	+ 2.24	— 0.01	−
481	6	2	2-3	7 38.23	65 25 53	65 31 49	1.0	— 2.19	+ 0.58	+ 1.53	+ 3.27	+ 0.02	+
497	9	—		7 5.83	10 0 21	10 52 5	0.5	+ 0.78	+ 0.98	+ 1.96	+ 0.22	+ 1.64	−
502	22	2	2-3	6 44.02	239 16 4	239 35 14	1.0	+ 2.12	— 0.26	+ 1.64	+ 2.97	— 0.19	+

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficients Δp_1	Δq_1	Δp_2	Δq_2	x_1	Δ
	1891			h m	° ′ ″	° ′ ″							
234	Sept. 18	2	2	6 14.87	66 40 24	66 40 41	1.0	— 0.35	— 2.51	— 2.69	+ 3.05	+ 0.04	+
399	Oct. 14	2-3	3	8 48.28	241 50 39	242 3 47	1.0	+ 2.41	— 0.23	+ 0.92	+ 3.73	— 0.10	+
423	16	2-3	3	9 13.24	66 5 32	66 6 17	1.0	— 1.71	— 1.70	— 0.83	+ 3.76	+ 0.02	+
445	19	2	3	9 6.79	64 33 30	64 38 38	1.0	— 1.02	+ 2.17	+ 2.62	+ 2.73	+ 0.06	
521	Nov. 30	3	3-4	6 27.64	246 47 29	246 42 26	0.0	— 2.09	+ 0.03	— 2.92	+ 1.54	+ 0.04	−
527	Dec. 8	2	3	6 18.12	65 10 17	65 15 37	1.0	— 2.03	— 0.18	+ 1.48	+ 2.90	+ 0.01	

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OSITION-ANGLES.

LL

k_1'	x_2	Δh_2	Δk_2	y_2	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	O—C	(O—C)× $s \sin 1''$	Differential aberration	Residuals	rot. Nr.
							′ ″	″	″	″	
.01	+ 0.11	— 0.16	— 0.06	— 0.02	0.00	+ 0.16	— 1 55	— 0.060	0.000	— 0.093	4
.06	— 0.08	+ 0.12	+ 0.05	0.00	+ 0.08	+ 0.13	— 4 3	— 0.367	0.000	— 0.080	19
.06	+ 0.09	— 0.02	— 0.09	+ 0.01	+ 0.06	+ 0.09	+ 2 14	+ 0.205	0.000	— 0.048	33
.09	+ 0.16	— 1.39	+ 0.32	+ 0.14	— 0.64	+ 1.36	— 122 29	— 0.165	— 0.002	+ 0.074	43
.11	+ 0.10	— 1.28	+ 0.33	+ 0.13	— 0.68	+ 1.29	— 172 20	— 0.232	— 0.001	+ 0.040	49
.02	+ 0.10	— 0.12	— 0.07	— 0.01	+ 0.01	+ 0.13	+ 0 19	+ 0.014	0.000	— 0.070	63
.06	— 0.09	— 0.32	+ 0.19	— 0.04	— 0.08	+ 0.35	— 42 48	— 0.488	— 0.001	+ 0.024	79
.07	— 0.18	— 0.06	+ 0.20	+ 0.02	+ 0.09	+ 0.19	+ 11 44	+ 0.440	0.000	— 0.003	102
.02	+ 0.09	— 0.09	— 0.07	— 0.01	+ 0.01	+ 0.11	+ 4 47	+ 0.274	0·000	+ 0.140	120
.03	— 0.04	— 0.23	+ 0.09	— 0.02	— 0.07	+ 0.24	— 38 2	— 0.552	— 0.001	— 0.077	141
.10	— 0.43	— 0.36	+ 0.49	+ 0.05	+ 0.13	+ 0.56	+ 37 17	+ 0.433	0.000	— 0.087	144
.03	+ 0.08	— 0.05	— 0.08	— 0.01	+ 0.03	+ 0.09	+ 2 43	+ 0.207	0.000	+ 0.004	163
.12	+ 0.13	+ 0.10	— 0.14	+ 0.01	+ 0.12	+ 0.17	+ 7 17	+ 0.416	0.000	+ 0.012	193
.04	— 0.04	+ 0.15	+ 0.04	0.00	+ 0.09	+ 0.15	— 5 46	— 0.450	0.000	— 0.125	203
.04	+ 0.08	— 0.03	— 0.08	— 0.01	+ 0.04	+ 0.08	+ 2 51	+ 0.238	0.000	— 0.006	220
.02	— 0.21	— 0.34	+ 0.20	+ 0.04	+ 0.01	+ 0.36	— 39 1	— 0.552	— 0.001	— 0.005	248
.06	— 0.05	+ 0.10	+ 0.06	0.00	+ 0.08	+ 0.12	— 0 40	— 0.063	0.000	+ 0.150	268
.04	+ 0.08	— 0.02	— 0.08	— 0.01	+ 0.04	+ 0.07	+ 4 8	+ 0.347	0.000	+ 0.076	293
.03	+ 0.02	— 0.11	— 0.03	— 0.01	— 0.06	+ 0.11	— 26 11	— 0.335	0.000	— 0.168	299
.20	+ 1.62	+ 3.36	— 1.29	+ 0.08	+ 2.24	+ 3.59	+ 289 40	+ 0.843	— 0.007	+ 0.487	328
.06	+ 0.08	+ 0.01	— 0.08	— 0.01	+ 0.05	+ 0.08	+ 2 46	+ 0.226	0.000	— 0.087	332
.04	+ 0.03	— 0.10	— 0.05	— 0.01	— 0.07	+ 0.11	— 10 44	— 0.137	0.000	— 0.080	334
.07	— 0.06	+ 0.05	+ 0.07	0.00	+ 0.07	+ 0.09	— 1 21	— 0.137	0.000	— 0.141	346
.03	+ 0.05	— 0.06	— 0.07	+ 0.01	— 0.02	+ 0.09	+ 1 59	+ 0.082	0.000	— 0.122	368
.04	+ 0.03	— 0.07	— 0.05	— 0.01	— 0.05	+ 0.08	+ 1 32	+ 0.025	0.000	+ 0.033	384
.04	0.00	+ 0.07	+ 0.05	0.00	+ 0.06	+ 0.09	— 0 1	— 0 001	0.000	+ 0.116	465
.03	+ 0.04	0.00	— 0.06	0.00	+ 0.02	+ 0.05	+ 5 56	+ 0.377	0.000	+ 0.092	481
.00	+ 2.28	+ 2.71	— 0.70	— 0.13	+ 1.63	+ 2.77	+ 51 44	+ 0.115	+ 0.002	— 0.272	497
.06	— 0.33	— 0.23	+ 0.25	+ 0.02	+ 0.18	+ 0.33	+ 19 10	+ 0.322	— 0.001	— 0.146	502

NLAY

k_1	x_2	Δh_2	Δk_2	y_2	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	O—C	(O—C)× $s \sin 1''$	Differential aberration	Residuals	rot. Nr.
							′ ″	″	″	″	
.04	+ 0.07	— 0.12	— 0.07	+ 0.01	— 0.03	+ 0.13	+ 0 17	+ 0.009	0.000	— 0.102	234
.06	— 0.22	— 0.11	+ 0.19	— 0.02	+ 0.11	+ 0.21	+ 13 8	+ 0.416	0.000	— 0 078	399
.03	+ 0.04	— 0.05	— 0.06	+ 0.01	— 0.01	+ 0.07	+ 0 45	+ 0.036	0.000	— 0.197	423
.07	+ 0.08	+ 0.04	— 0.08	— 0.01	+ 0.06	+ 0.08	+ 5 8	+ 0.333	0.000	+ 0.002	445
.02	+ 0.06	+ 0.10	+ 0.04	0.00	+ 0.06	+ 0.10	— 5 3	— 0.292	0.000	[— 0.126]	521
.02	+ 0.02	0.00	— 0.05	0.00	0.00	+ 0.04	+ 5 20	+ 0.241	0.000	— 0.33	527

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TABLE VI. SATELLITES I and II

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficients μ	z_G	x_1	Δh_1	Δk_1	
	1891			h m	"	"								
10	Aug. 17	2	2	10 15.84	174.754	174.812	c	1.0	— 1.75	— 0.57	+ 0.82	— 1.15	+ 1.36	
26	19	2-3	3	10 27.32	22.230	22.633	ψ	1.0	— 0.22	0.00	— 0.03	— 1.17	+ 0.42	
28	20	2-3	3	7 56.92	21.916	22.566	ψ	1.0	— 0.22	0.00	— 0.06	— 1.12	+ 0.34	
62	24	2-3	2-3	9 2.11	182.147	182.100	c	1.0	— 1.82	— 0.55	+ 1.01	— 1.68	+ 1.58	
72	25	3	3	8 5.86	474.098	474.607	c	0.5	— 4.74	— 0.21	+ 1.03	— 1.36	+ 1.46	
101	29	2-3	2-3	8 39.00	251.654	251.808	c	1.0	— 2.52	— 0.40	+ 0.03	+ 2.16	— 0.48	
123	31	2-3	2-3	9 6.27	182.628	182.797	c	1.0	— 1.83	— 0.55	+ 1.02	— 2.01	+ 1.46	
133	Sept. 1	—		7 50.60	440.018	440.453	c	1.0	— 4.40	— 0.23	+ 1.16	— 1.75	+ 1.53	
156	5	2	2-3	8 35.96	240.515	240.534	c	1.0	— 2.41	— 0.41	0.00	+ 1.89	— 0.27	
166	7	1-2	2-3	7 26.74	197.920	197.862	c	1.0	— 1.98	— 0.51	+ 0.61	— 2.35	+ 0.89	
188	10	2	2-3	6 43.86	141.105	141.907	c	1.0	— 1.41	— 0.70	+ 1.24	— 1.29	+ 1.34	
206	13	2	3	7 15.08	310.381	309.949	c	1.0	— 3.10	— 0.32	— 0.17	+ 1.81	— 0.24	
219	14	3	3 & 3-4	6 24.85	196.903	196.820	c	0.5	— 1.97	— 0.51	+ 0.32	— 2.15	+ 0.38	
251	19	2	3	6 11.41	225.436	225.176	c	1.0	— 2.25	— 0.44	— 0.80	+ 1.09	— 0.76	
271	20	1-2	2	6 18.75	379.083	378.834	c	1.0	— 3.79	— 0.26	— 0.39	+ 1.33	— 0.32	
292	21	1	1-2	6 49.88	178.932	178.977	c	1.0	— 1.79	— 0.56	+ 0.34	— 1.94	+ 0.22	
302	22	2	2-3	6 28.34	255.136	255.675	c	1.0	— 2.55	— 0.39	+ 0.68	— 2.26	+ 0.51	
341	29	3-4	4	6 9.29	190.802	191.158	c	0.25	— 1.91	— 0.52	+ 0.54	— 1.96	+ 0.22	
355	Oct. 4	3	3	6 29.23	428.764	428.783	c	0.5	— 4.29	— 0.23	— 1.07	+ 1.08	— 0.85	
367	9	3	3	8 59.46	221.074	220.539	c	0.5	— 2.21	— 0.45	— 1.78	+ 1.64	— 1.36	
381	10	2-3	3	8 57.49	220.985	220.400	c	1.0	— 2.21	— 0.45	— 1.43	+ 1.21	— 1.11	
389	13	1-2	1-2	9 9.18	145.534	146.044	c	1.0	— 1.46	— 0.68	+ 0.75	— 1.82	+ 0.14	
464	Nov. 5	2-3	3-4	7 5.94	171.060	171.424	c	1.0	— 1.71	— 0.58	+ 0.93	— 1.39	+ 0.12	
480	6	2	2-3	7 27.86	140.115	140.336	c	1.0	— 1.40	— 0.71	+ 1.30	— 1.88	+ 1.20	
516	27	2-3	2-3	6 43.64	301.726	302.208	c	1.0	— 3.02	— 0.33	+ 1.04	— 0.95	+ 0.48	
550	Dec. 14	3	3-4	7 16.92	376.104	375.956	c	0.5	— 3.76	— 0.27	— 1.41	+ 1.48	— 0.16	

OBSERV

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficients μ	z_F	x_1	Δh_1	Δk_1	
	1891			h m	"	"								
241	Sept. 18	2	3	6 53.54	365.163	364.686	c	1.0	— 3.65	— 0.27	— 1.19	+ 1.16	— 1.17	
365	Oct. 6	2-3	3	6 32.41	144.731	145.287	ψ	1.0	— 1.45	0.00	+ 0.56	— 1.70	+ 0.14	
406	14	2-3	3-4	9 30.56	403.929	404.017	c	1.0	— 4.04	— 0.25	+ 1.25	— 2.23	+ 0.50	
430	16	3	3-4	9 52.61	170.662	170.310	c	0.5	— 1.71	— 0.59	— 1.88	+ 1.78	— 1.31	
434	17	2-3	3-4	8 32.42	228.743	228.154	c	1.0	— 2.29	— 0.44	— 1.78	+ 1.52	— 1.28	
442	19	2	3	8 39.50	56.144	56.047	c	1.0	— 0.56	— 1.78	+ 0.89	— 1.08	+ 0.48	
524	Nov. 30	v. bad		6 43.17	322.260	321.966	c	0.0	— 3.22	— 0.31	— 1.83	+ 1.92	— 0.44	
526	Dec. 8	2	3	6 6.92	81.787	81.352	c	1.0	— 0.82	— 1.23	— 1.88	+ 1.87	— 0.50	

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STANCES.

LL

	Δk_3	y_3	$\Delta\varepsilon_1$	$\Delta\varepsilon_3$	Δp_1	Δq_1	Δp_3	Δq_3	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
01	− 3.50	+ 0.72	— 1.46	+ 3.85	+ 0.01	+ 0.07	— 0.02	— 0.17	+ 0.058	+ 0.002	— 0.038	10
21	— 1.68	+ 0.52	+ 0.50	— 5.31	+ 0.66	— 0.71	— 2.46	— 0.17	+ 0.403	+ 0.010	— 0.194	26
01	— 3.33	+ 0.61	— 0.56	— 5.14	— 0.99	+ 0.95	— 2.29	— 2.62	+ 0.650	+ 0.010	+ 0.079	28
52	— 3.79	— 0.35	— 2.22	+ 4.68	+ 0.03	+ 0.05	— 0.05	— 0.15	— 0.047	— 0.003	— 0.037	62
88	— 1.47	+ 0.38	+ 1.78	— 0.21	0.00	— 0.02	+ 0.02	— 0.04	+ 0.509	— 0.004	+ 0.043	72
92	+ 0.75	— 0.54	+ 2.08	— 2.70	0.00	0.00	0.00	0.00	+ 0.154	+ 0.004	+ 0.057	101
95	— 3.87	+ 0.29	— 2.47	+ 5.18	+ 0.05	+ 0.05	— 0.07	— 0.16	+ 0.169	+ 0.009	+ 0.164	123
74	— 1.81	+ 0.16	+ 2.24	+ 0.63	— 0.02	— 0.03	+ 0.03	— 0.08	+ 0.435	— 0.006	— 0.049	133
57	+ 0.82	+ 0.22	+ 1.63	— 1.97	0.00	0.00	0.00	0.00	+ 0.019	+ 0.002	— 0.038	156
60	— 3.82	— 0.79	— 2.50	+ 5.80	+ 0.07	+ 0.02	— 0.11	— 0.15	— 0.058	+ 0.003	— 0.046	166
87	— 2.74	— 0.76	+ 1.57	— 6.47	+ 0.03	+ 0.14	— 0.33	— 0.14	+ 0.803	+ 0.011	+ 0.054	188
62	+ 1.22	+ 0.50	— 1.51	— 5.49	— 0.08	+ 0.04	— 0.23	+ 0.02	— 0.432	— 0.007	— 0.129	206
11	— 3.56	— 0.72	— 2.05	+ 6.15	+ 0.09	— 0.02	— 0.16	— 0.15	— 0.083	+ 0.006	— 0.146	219
87	+ 1.47	+ 0.38	— 0.44	+ 0.13	0.00	0.00	0.00	0.00	— 0.260	0.000	— 0.013	251
75	+ 0.78	+ 0.77	— 0.61	— 4.88	— 0.04	+ 0.04	— 0.14	+ 0.04	— 0.249	— 0.008	— 0.068	271
10	— 3.25	— 0.66	— 1.71	+ 6.28	+ 0.10	— 0.04	— 0.21	— 0.16	+ 0.045	+ 0.009	— 0.071	292
00	— 2.92	— 0.30	+ 2.24	+ 3.10	— 0.09	0.00	— 0.01	— 0.23	+ 0.539	— 0.003	+ 0.059	302
8	— 3.16	+ 0.29	+ 1.77	+ 3.78	— 0.11	+ 0.04	— 0.06	— 0.30	+ 0.356	+ 0.001	— 0.120	341
4	+ 0.37	— 0.31	+ 0.67	— 3.65	0.00	+ 0.02	— 0.05	+ 0.03	+ 0.019	— 0.003	+ 0.094	355
0	+ 3.34	— 0.68	+ 2.01	+ 5.72	— 0.07	— 0.11	— 0.23	— 0.24	— 0.535	— 0.006	— 0.124	367
3	+ 2.13	— 0.47	— 1.26	+ 1.77	0.00	+ 0.01	0.00	— 0.04	— 0.585	+ 0.001	— 0.119	381
1	— 3.22	+ 0.38	+ 1.58	+ 4.31	— 0.14	+ 0.07	— 0.14	— 0.36	+ 0.510	+ 0.002	+ 0.025	389
6	— 0.24	— 0.49	+ 0.93	— 2.80	+ 0.02	— 0.02	— 0.05	+ 0.04	+ 0.364	+ 0.002	— 0.061	464
1	— 1.74	— 0.64	— 1.75	— 5.66	— 0.16	+ 0.05	— 0.40	— 0.09	+ 0.223	+ 0.006	— 0.104	480
9	— 0.62	+ 0.34	+ 0.11	— 4.36	— 0.01	+ 0.03	— 0.09	+ 0.02	+ 0.482	+ 0.007	— 0.032	516
1	+ 0.78	+ 0.15	— 1.27	— 0.77	— 0.01	0.00	— 0.02	+ 0.02	— 0.148	+ 0.003	+ 0.182	550

LAY

	Δk_3	y_3	$\Delta\varepsilon_1$	$\Delta\varepsilon_3$	Δp_1	Δq_1	Δp_3	Δq_3	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
1	+ 3.67	+ 0.40	+ 1.21	+ 4.94	0.00	— 0.07	— 0.08	— 0.17	— 0.477	— 0.006	— 0.069	241
5	— 3.26	+ 0.72	+ 1.36	+ 4.24	— 0.13	+ 0.08	— 0.13	— 0.37	+ 0.556	+ 0.004	+ 0.064	365
0	— 0.92	+ 0.43	— 2.24	— 0.68	— 0.02	0.00	— 0.03	+ 0.05	+ 0.088	— 0.005	+ 0.049	406
0	+ 3.12	— 0.60	+ 2.14	+ 5.82	— 0.11	— 0.13	— 0.32	— 0.28	— 0.352	— 0.006	+ 0.230	430
3	+ 2.48	— 0.30	— 1.84	+ 2.55	+ 0.01	+ 0.01	0.00	— 0.04	— 0.589	+ 0.003	+ 0.100	434
	— 1.65	+ 0.72	+ 0.07	+ 5.82	+ 0.13	— 0.33	— 0.89	— 0.15	— 0.097	+ 0.012	+ 0.099	442
	+ 0.36	— 0.43	— 1.93	— 2.21	— 0.03	0.00	— 0.06	+ 0.06	— 0.294	+ 0.004	[+ 0.215]	524
	+ 1.09	— 0.63	+ 1.90	— 4.68	+ 0.08	0.00	— 0.21	0.00	— 0.435	— 0.003	+ 0.067	526

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TABLE VII. SATELLITES I and

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	OBSERV. Δp_1	OBSERV. Δq_1	OBSERV. Δp_3	OBSERV. Δq_3	Coeff. x_1
	1891			h m	° ′ ″	° ′ ″						
9	Aug. 17	2	2	10 16.36	63 32 1	63 27 8	1.0	— 0.23	— 2.51	+ 0.78	+ 6.35	+ 0.09
25	19	2-3	3	10 26.99	87 35 27	87 28 43	1.0	+ 1.59	— 1.70	— 5.93	— 0.40	+ 0.09
27	20	2-3	3	7 55.79	97 40 36	96 49 5	1.0	— 1.53	+ 1.48	— 3.56	— 4.07	+ 0.12
61	24	2-3	2-3	9 1.93	63 40 34	63 28 38	1.0	— 1.33	— 2.17	+ 1.92	+ 6.14	+ 0.08
71	25	3	3	8 5.20	64 41 16	64 39 13	0.5	+ 0.67	+ 2.45	— 3.32	+ 5.54	+ 0.07
100	29	2-3	2-3	8 39.30	245 12 44	245 8 32	1.0	+ 2.54	— 0.31	— 5.18	+ 3.87	— 0.04
122	31	2-3	2-3	9 6.33	63 40 9	63 30 26	1.0	— 1.85	— 1.76	+ 2.74	+ 5.86	+ 0.07
132	Sept. 1	—		7 49.30	64 27 28	64 24 47	1.0	+ 1.40	+ 2.13	— 2.49	+ 5.98	+ 0.07
155	5	2	2-3	8 35.60	245 11 28	245 7 47	1.0	+ 2.36	— 0.99	— 4.60	+ 4.56	— 0.05
165	7	1-2	2-3	7 26.19	63 39 12	63 32 2	1.0	— 2.48	— 0.63	+ 3.83	+ 5.21	+ 0.05
187	10	2	2-3	6 43.87	68 30 30	68 33 28	1.0	+ 0.49	+ 2.50	— 5.97	— 2.54	0.00
205	13	2	3	7 14.96	247 24 6	247 22 12	1.0	— 2.27	+ 1.14	— 6.42	+ 0.55	— 0.04
218	14	3	3 & 3-4	6 24.75	63 25 11	63 20 24	0.5	— 2.51	+ 0.45	+ 4.66	+ 4.45	+ 0.05
250	19	2	3	6 11.49	245 23 40	245 18 38	1.0	+ 0.69	— 2.44	— 2.69	+ 5.85	— 0.07
270	20	1-2	2	6 18.42	246 42 8	246 39 55	1.0	— 1.61	+ 1.94	— 6.20	+ 1.63	— 0.05
291	21	1	1-2	6 49.24	63 8 57	63 3 11	1.0	— 2.35	+ 0.95	+ 5.12	+ 3.83	+ 0.05
301	22	2	2-3	6 28.44	63 25 19	63 19 45	1.0	+ 2.52	— 0.08	+ 0.40	+ 6.39	+ 0.06
340	29	3-4	4	6 7.63	62 48 23	62 44 16	0.25	+ 2.34	— 0.86	+ 1.32	+ 6.19	+ 0.06
354	Oct. 4	3	3	6 28.96	246 10 20	246 8 28	1.0	— 0.36	+ 2.45	— 5.42	+ 3.16	— 0.05
366	9	3	3	6 58.90	242 37 10	242 31 0	0.5	+ 1.28	+ 2.09	+ 4.25	+ 4.55	— 0.14
380	10	2-3	3	8 56.99	245 22 20	245 16 59	1.0	— 0.32	— 2.43	— 0.77	+ 6.16	— 0.06
388	13	1-2	1-2	9 9.16	62 7 20	62 1 57	1.0	+ 2.17	— 1.05	+ 2.23	+ 5.72	+ 0.07
463	Nov. 5	2-3	3-4	7 6.32	66 28 29	66 23 58	1.0	+ 1.66	— 1.54	— 4.53	+ 3.57	+ 0.01
479	6	2	2-3	7 27.50	69 56 10	69 54 23	1.0	— 2.17	+ 0.64	— 5.60	— 1.27	— 0.09
515	27	2-3	2-3	6 42.52	66 41 49	66 41 8	1.0	— 0.75	+ 1.98	— 5.25	+ 1.13	0.00
549	Dec. 14	3	3-4	7 16.07	245 49 59	245 48 22	0.5	— 1.81	+ 0.84	— 2.86	+ 4.20	+ 0.02

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	OBSERV. Δp_1	OBSERV. Δq_1	OBSERV. Δp_3	OBSERV. Δq_3	Coeff. x_1
	1891			h m	° ′ ″	° ′ ″						
240	Sept. 18	2	3	6 53.27	243 47 56	243 44 29	1.0	+ 0.11	+ 2.53	+ 2.64	+ 5.89	+ 0.10
364	Oct. 6	2-3	3	6 32.75	62 6 28	61 57 58	1.0	+ 2.07	— 1.32	+ 2.03	+ 5.90	+ 0.07
405	14	2-3	3-4	9 30.41	66 16 18	66 11 29	1.0	— 2.43	— 0.02	— 3.05	+ 5.33	+ 0.01
429	16	3	3-4	9 52.06	241 46 33	241 40 35	0.5	+ 1.53	+ 1.85	+ 4.61	+ 4.01	— 0.17
433	17	2-3	3-4	8 32.43	245 23 40	245 17 37	1.0	— 1.07	— 2.15	+ 0.16	+ 6.10	— 0.05
441	19	2	3	8 39.76	57 15 17	57 7 25	0.0	— 0.86	+ 2.21	+ 5.90	+ 0.98	+ 0.18
523	Nov. 30	v. bad.		6 43.41	246 34 29	246 33 10	0.0	— 2.09	— 0.05	— 3.98	+ 3.51	+ 0.03
525	Dec. 8	2	3	6 6.83	247 53 17	247 55 18	1.0	+ 2.04	+ 0.12	— 5.16	0.00	+ 0.08

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SITION-ANGLES.

x_3	Δh_3	Δk_3	y_3	$\Delta\varepsilon_1$	$\Delta\varepsilon_3$	O—C	(O—C) × $s \sin 1''$	Differential aberration	Residuals	rot. Nr.
						′ ″	″	″	″	
— 0.21	+ 0.04	— 0.24	+ 0.04	— 0.10	+ 0.25	— 4 53	— 0.247	0.000	+ 0.137	9
— 0.22	— 2.27	+ 0.62	— 0.21	— 0.29	+ 2.31	— 6 44	— 0.644	— 0.004	+ 0.030	25
+ 1.00	— 2.62	+ 2.02	— 0.39	+ 0.45	+ 3.25	— 51 31	— 0.328	— 0.006	— 0.243	27
— 0.21	+ 0.05	— 0.23	0.00	— 0.09	+ 0.24	— 11 56	— 0.632	0.000	— 0.201	61
— 0.19	— 0.08	— 0.17	— 0.02	+ 0.07	+ 0.19	— 2 3	— 0.283	0.000	+ 0.119	71
+ 0.16	+ 0.13	+ 0.13	+ 0.03	— 0.05	+ 0.18	— 4 12	— 0.307	0.000	— 0.158	100
— 1.21	+ 0.08	— 0.23	0.00	— 0.09	+ 0.24	— 9 43	— 0.517	0.000	— 0.074	122
— 0.20	— 0.05	— 0.19	— 0.02	+ 0.07	+ 0.20	— 2 41	— 0.343	0.000	+ 0.052	132
+ 0.16	+ 0.11	+ 0.14	+ 0.02	— 0.06	+ 0.18	— 3 41	— 0.259	0.000	— 0.067	155
— 0.20	+ 0.10	— 0.22	— 0.04	— 0.05	+ 0.24	— 7 10	— 0.413	0.000	+ 0.043	165
+ 0.05	— 0.38	+ 0.08	+ 0.03	— 0.03	+ 0.38	+ 2 58	+ 0.122	— 0.001	+ 0.008	187
+ 0.05	+ 0.30	+ 0.04	— 0.01	+ 0.11	+ 0.30	— 1 54	— 0.171	+ 0.001	— 0.040	205
— 0.21	+ 0.14	— 0.21	— 0.03	— 0.03	+ 0.25	— 4 47	— 0.274	0.000	+ 0.154	218
+ 0.16	+ 0.08	+ 0.16	— 0.02	— 0.07	+ 0.18	— 5 2	— 0.329	0.000	— 0.010	250
+ 0.06	+ 0.22	+ 0.08	— 0.04	+ 0.08	+ 0.23	— 2 13	— 0.245	0.000	— 0.038	270
— 0.23	+ 0.18	— 0.22	— 0.02	— 0.02	+ 0.28	— 5 46	— 0.300	+ 0.001	+ 0.081	291
— 0.26	+ 0.07	— 0.26	0.00	+ 0.05	+ 0.26	— 5 34	— 0.414	0.000	— 0.095	301
— 0.31	+ 0.13	— 0.29	+ 0.03	+ 0.04	+ 0.31	— 4 7	— 0.229	0.000	+ 0.062	340
+ 0.07	+ 0.14	+ 0.10	— 0.01	+ 0.06	+ 0.17	— 1 52	— 0.233	0.000	+ 0.047	354
+ 0.33	— 0.24	+ 0.27	— 0.04	+ 0.15	+ 0.36	— 6 10	— 0.396	— 0.001	— 0.044	366
+ 0.14	+ 0.03	+ 0.16	— 0.01	— 0.06	+ 0.16	— 5 21	— 0.343	0.000	+ 0.048	380
— 0.37	+ 0.21	— 0.32	+ 0.02	+ 0.05	+ 0.38	— 5 23	— 0.228	+ 0.001	+ 0.033	388
0.00	— 0.12	— 0.10	+ 0.01	— 0.06	+ 0.15	— 4 31	— 0.225	0.000	— 0.086	463
+ 0.32	— 0.44	+ 0.07	+ 0.05	+ 0.16	+ 0.44	— 1 47	— 0.073	— 0.001	— 0.039	479
+ 0.09	— 0.15	— 0.05	0.00	+ 0.05	+ 0.15	— 0 41	— 0.060	0.000	+ 0.096	515
— 0.03	+ 0.08	+ 0.10	+ 0.01	+ 0.04	+ 0.12	— 1 37	— 0.177	0.000	+ 0.199	549

LAY

x_3	Δh_3	Δk_3	y_3	$\Delta\varepsilon_1$	$\Delta\varepsilon_3$	O—C	(O—C) × $s \sin 1''$	Differential aberration	Residuals	rot. Nr.
						′ ″	″	″	″	
— 0.23	— 0.09	+ 0.23	0.00	+ 0.10	+ 0.25	— 3 27	— 0.366	0.000	+ 0.108	240
— 0.37	+ 0.20	— 0.33	+ 0.06	+ 0.03	+ 0.38	— 8 30	— 0.359	+ 0.001	— 0.089	364
— 0.08	— 0.10	— 0.13	— 0.02	+ 0.05	+ 0.16	— 4 49	— 0.566	0.000	— 0.106	405
+ 0.40	— 0.33	+ 0.30	— 0.03	+ 0.18	+ 0.44	— 5 58	— 0.297	0.000	+ 0.010	429
+ 0.13	+ 0.02	+ 0.15	+ 0.01	— 0.05	+ 0.15	— 6 3	— 0.403	0.000	+ 0.023	433
— 0.61	+ 0.79	— 0.31	+ 0.10	+ 0.07	+ 0.83	— 7 52	— 0.128	+ 0.002	[— 0.002]	441
— 0.05	+ 0.12	+ 0.09	+ 0.01	+ 0.05	+ 0.15	— 1 19	— 0.123	0.000	[+ 0.189]	523
— 0.21	+ 0.25	+ 0.01	+ 0.04	— 0.09	+ 0.25	+ 2 1	+ 0.047	0.000	— 0.054	525

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TABLE VIII. SATELLITES I and I

OBSERV

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coeffic μ	Coeffic z_G	Coeffic x_1	Coeffic Δh_1	Coeffic Δk_1	Coeffic
	1891			h m	"	"								
1	Aug. 17	2	2	9 4.81	23.090	22.321	ψ	1.0	— 0.23	0.00	— 0.39	+ 0.66	— 0.70	-
18	19	2-3	3	9 6.64	519.420	519.377	c	1.0	— 5.19	— 0.19	— 0.17	+ 1.11	— 0.60	-
38	20	2	3	9 27.55	417.816	417.836	c	1.0	— 4.18	— 0.24	+ 0.28	+ 1.63	— 0.30	-
60	24	2-3	2-3	8 51.29	440.178	440.281	c	1.0	— 4.40	— 0.23	— 1.00	+ 1.71	— 1.57	-
83	25	2-3	2-3	9 12.76	31.965	31.353	ψ	1.0	— 0.32	0.00	— 0.80	+ 0.95	— 1.11	-
93	29	2-3	3	7 45.92	724.356	724.290	c	1.0	— 7.24	— 0.14	— 0.11	— 2.03	+ 0.37	+
125	31	2-3	2-3	9 15.26	547.041	547.161	c	1.0	— 5.47	— 0.18	+ 1.04	— 1.99	+ 1.48	+
131	Sept. 1	—		7 38.74	519.370	520.345	c	0.0	— 5.19	— 0.19	+ 1.15	— 1.78	+ 1.52	+
151	5	3	3-4	7 48.10	323.206	322.893	c	0.5	— 3.23	— 0.31	+ 0.01	+ 1.74	— 0.24	-
196	10	2	2-3	7 36.68	172.420	171.716	c	1.0	— 1.72	— 0.58	— 1.12	+ 1.18	— 1.22	-
214	13	2	3	8 5.95	301.038	301.134	c	1.0	— 3.01	— 0.33	+ 0.20	— 1.97	+ 0.28	+
227	14	1-2 & 2	3-4 & 4	7 15.83	633.641	633.634	c	1.0	— 6.34	— 0.16	+ 0.45	— 2.26	+ 0.51	+
259	19	1-2	2	7 4.83	299.676	300.442	c	1.0	— 3.00	— 0.33	+ 0.64	— 1.12	+ 0.60	+
267	20	1-2	2	5 55.85	220.818	220.303	c	1.0	— 2.21	— 0.45	— 0.43	+ 1.28	— 0.37	-
296	21	1	1-2	7 13.00	220.084	219.859	c	1.0	— 2.20	— 0.45	— 0.37	+ 2.00	— 0.25	-
310	22	2	2-3	7 11.34	556.746	556.275	c	1.0	— 5.57	— 0.18	— 0.81	+ 2.31	— 0.63	-
345	Oct. 4	3	3	5 30.21	354.637	355.255	c	0.5	— 3.55	— 0.28	+ 1.22	— 1.11	+ 1.00	+
377	9	2	2-3	9 58.83	650.301	650.233	c	1.0	— 6.50	— 0.15	— 1.70	+ 1.46	— 1.33	-
398	13	3	3	10 8.14	396.771	396.193	c	1.0	— 3.97	— 0.25	— 0.87	+ 1.98	— 0.21	-
474	Nov. 5	2-3	3-4	8 43.68	461.728	462.100	c	1.0	— 4.62	— 0.22	+ 1.08	— 1.66	+ 0.10	+
486	6	3 & 3-4	3 & 3-4	8 11.97	541.063	541.546	c	0.5	— 5.41	— 0.18	+ 1.43	— 1.98	+ 0.27	+
512	22	3	3-4	7 52.26	561.259	561.619	c	0.5	— 5.61	— 0.18	+ 1.82	— 1.99	+ 0.48	+
514	27	2-3	2-3	6 31.80	198.647	198.094	c	1.0	— 1.99	— 0.51	— 1.05	+ 0.94	— 0.50	-
548	Dec. 14	2	3	7 1.27	431.040	430.564	c	1.0	— 4.31	— 0.23	— 1.37	+ 1.44	— 0.15	-

OBSERV

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coeffic μ	Coeffic z_F	Coeffic x_1	Coeffic Δh_1	Coeffic Δk_1	Coeffic
	1891			h m	"	"								
84	Aug. 27	3-4	3	7 40.09	463.503	463.311	c	0.25	— 4.64	— 0.22	+ 0.31	— 1.11	+ 0.62	+
361	Oct. 6	2-3	3	5 54.13	57.247	56.280	ψ	1.0	— 0.57	0.00	— 0.53	+ 1.55	— 0.14	-
404	14	2-3	3	9 16.24	36.214	35.967	ψ	1.0	— 0.36	0.00	— 1.17	+ 2.12	— 0.45	-
426	16	2-3	3	9 24.66	297.119	296.979	c	1.0	— 2.97	— 0.34	+ 1.90	— 1.86	+ 1.29	+
436	17	2-3	3-4	8 46.41	601.801	602.148	c	1.0	— 6.02	— 0.17	+ 1.75	— 1.48	+ 1.27	+
450	19	2	3	9 35.95	743.596	744.079	c	1.0	— 7.44	— 0.13	+ 0.80	— 1.16	+ 0.35	+
530	Dec. 8	2	3	6 30.99	560.258	560.333	c	1.0	— 5.60	— 0.18	+ 1.92	— 1.89	+ 0.56	+

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STANCES.

L

	y_4	$\Delta\varepsilon_1$	$\Delta\varepsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr
								"	"	"	
0	+ 0.07	+ 0.85	+ 5.56	— 0.59	— 2.11	— 9.12	— 3.70	— 0.769	— 0.006	— 0.040	1
9	+ 0.40	— 0.08	+ 9.21	+ 0.03	— 0.05	— 0.11	— 0.24	— 0.043	— 0.012	+ 0.246	18
8	+ 0.70	+ 1.24	+ 6.06	— 0.05	+ 0.03	— 0.01	— 0.25	+ 0.020	— 0.003	+ 0.039	38
2	+ 0.25	+ 2.25	— 9.18	+ 0.03	+ 0.05	— 0.27	+ 0.03	+ 0.103	— 0.005	+ 0.090	60
1	+ 0.07	— 1.20	— 8.78	— 0.15	— 1.51	— 6.47	— 1.87	— 0.612	— 0.011	+ 0.037	83
7	— 0.46	— 1.87	+ 3.45	0.00	0.00	0.00	0.00	— 0.066	— 0.003	— 0.010	93
1	— 0.14	— 2.45	— 5.15	— 0.04	— 0.04	— 0.24	+ 0.16	+ 0.120	— 0.004	— 0.144	125
4	— 0.03	+ 2.27	— 8.36	+ 0.03	+ 0.04	— 0.21	+ 0.06	+ 0.975	+ 0.002	[+ 0.058]	131
6	— 0.11	+ 1.39	+ 8.32	— 0.09	+ 0.05	— 0.15	— 0.43	— 0.313	— 0.007	— 0.173	151
7	+ 0.30	— 1.30	— 9.97	— 0.02	— 0.26	— 1.14	+ 0.01	— 0.704	— 0.012	— 0.199	196
5	+ 0.37	+ 1.76	+ 8.87	— 0.11	+ 0.04	— 0.22	— 0.48	+ 0.096	+ 0.007	— 0.112	214
6	+ 0.67	— 2.23	+ 5.80	+ 0.01	0.00	0.00	— 0.04	— 0.007	— 0.001	+ 0.120	227
6	+ 0.13	+ 0.12	— 11.07	— 0.05	+ 0.12	— 0.58	— 0.13	+ 0.766	+ 0.017	+ 0.024	259
3	— 0.14	— 0.47	+ 11.26	+ 0.10	— 0.13	— 0.63	— 0.40	— 0.515	— 0.018	+ 0.176	267
7	— 0.32	+ 1.81	+ 9.80	— 0.17	+ 0.06	— 0.45	— 0.65	— 0.225	— 0.010	+ 0.053	296
2	— 0.46	— 2.35	+ 7.03	+ 0.02	0.00	— 0.02	— 0.08	— 0.471	0.000	— 0.019	310
5	— 0.45	— 0.96	— 7.26	0.00	— 0.09	— 0.36	+ 0.17	+ 0.618	+ 0.006	+ 0.049	345
8	+ 0.66	+ 1.79	+ 5.05	— 0.01	— 0.02	— 0.01	— 0.12	— 0.068	0.000	— 0.047	377
8	+ 0.27	— 1.83	— 9.30	— 0.09	+ 0.03	— 0.42	+ 0.05	— 0.578	— 0.008	— 0.046	398
5	+ 0.75	+ 1.40	— 2.75	+ 0.01	— 0.01	— 0.05	+ 0.06	+ 0.372	— 0.002	— 0.242	474
0	+ 0.54	— 1.90	— 6.05	— 0.05	+ 0.01	— 0.18	+ 0.11	+ 0.483	0.000	+ 0.003	486
5	— 0.40	— 2.01	— 3.44	— 0.03	0.00	— 0.10	+ 0.10	+ 0.360	— 0.004	+ 0.013	512
8	+ 0.26	— 0.17	+ 7.62	— 0.02	+ 0.07	— 0.17	— 0.29	— 0.553	— 0.010	— 0.124	514
8	— 0.06	— 1.21	+ 6.69	+ 0.02	— 0.01	— 0.04	— 0.09	— 0.476	— 0.006	— 0.029	548

LAY

	y_4	$\Delta\varepsilon_1$	$\Delta\varepsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr
								"	"	"	
1	— 0.32	+ 0.03	+ 9.89	+ 0.04	— 0.06	— 0.18	— 0.28	— 0.192	+ 0.014	— 0.208	84
4	+ 0.13	— 1.14	+ 10.76	+ 0.43	— 0.34	— 2.34	— 0.75	— 0.967	— 0.015	— 0.030	361
7	+ 0.13	+ 2.11	— 10.16	+ 0.69	— 0.02	— 2.94	— 0.77	— 0.247	— 0.009	— 0.016	404
4	— 0.34	— 2.21	+ 8.79	+ 0.04	+ 0.04	— 0.12	— 0.19	— 0.140	+ 0.004	+ 0.023	426
9	— 0.47	+ 1.78	+ 6.02	— 0.01	— 0.03	— 0.03	— 0.14	+ 0.347	+ 0.002	+ 0.025	436
2	— 0.33	— 0.26	— 1.76	— 0.01	+ 0.01	— 0.04	+ 0.06	+ 0.483	+ 0.001	+ 0.081	450
3	— 0.30	— 1.94	— 0.57	— 0.02	0.00	— 0.04	+ 0.07	+ 0.075	— 0.006	— 0.069	530

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TABLE IX. SATELLITES I and I

OBSERV

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coeffic Δp_1	Coeffic Δq_1	Coeffic Δp_4	Coeffic Δq_4	
	1891			h m	° ′ ″	° ′ ″						
2	Aug. 17	2	2	9 12.70	191 25 39	190 48 51	1.0	+ 0.37	+ 1.45	+ 6.23	+ 2.54	-
17	19	2-3	3	9 6.24	243 41 44	243 42 15	1.0	— 1.32	+ 2.15	+ 4.75	+ 10.25	-
37	20	2	3	9 28.42	243 46 25	243 46 32	1.0	+ 2.17	— 1.31	+ 0.56	+ 11.26	-
59	24	2-3	2-3	8 51.32	246 29 51	246 23 7	1.0	+ 1.38	+ 2.13	— 11.18	+ 1.18	-
82	25	2-3	2-3	9 13.00	281 56 54	281 32 7	1.0	— 0.21	— 2.03	— 8.67	— 2.50	+
92	29	2-3	3	7 45.78	65 11 3	65 9 18	1.0	— 2.48	+ 0.63	— 2.19	+ 11.22	+
124	31	2-3	2-3	9 15.20	66 37 11	66 32 15	1.0	— 1.81	— 1.80	— 9.40	+ 6.60	+
130	Sept. 1	—		7 38.41	66 15 52	66 9 36	1.0	+ 1.45	+ 2.09	— 11.10	+ 3.01	+
150	5	3	3-4	7 48.00	242 58 9	242 59 57	0.5	+ 2.23	— 1.26	+ 3.77	+ 10.76	-
161	7	1-2	2-3	7 1.06	244 58 53	244 58 34	1.0	+ 2.52	+ 0.47	— 4.52	+ 10.40	-
195	10	2	2-3	7 36.38	251 7 10	250 55 20	1.0	— 0.16	— 2.53	— 11.23	+ 0.08	+
213	13	2	3	8 5.68	62 40 54	62 42 45	1.0	+ 2.39	— 0.85	+ 4.77	+ 10.29	+
226	14	1-2 & 2	3-4 & 4	7 15.52	65 8 58	65 7 4	1.0	— 2.55	+ 0.13	+ 0.82	+ 11.36	+
258	19	1-2	2	7 3.82	68 25 37	68 16 47	1.0	— 1.00	+ 2.33	— 11.13	— 2.41	+
266	20	1-2	2	5 55.95	241 41 57	241 46 36	1.0	— 1.50	+ 2.03	+ 9.56	+ 6.11	-
295	21	1	1-2	7 12.95	241 28 54	241 35 42	1.0	+ 2.40	— 0.82	+ 6.43	+ 9.28	-
309	22	2	2-3	7 10.36	245 4 34	245 5 3	1.0	— 2.51	— 0.19	+ 2.56	+ 10.79	-
344	Oct. 4	3	3	5 30.66	67 41 54	67 35 17	1.0	+ 0.06	— 2.47	— 10.09	+ 4.74	+
376	9	2	2-3	9 58.98	245 3 41	245 4 5	1.0	+ 0.96	+ 2.25	+ 0.74	+ 10.88	-
397	13	3	3	10 4.69	247 58 43	247 51 58	0.5	— 2.31	+ 0.72	— 10.66	+ 1.19	+
473	Nov. 5	2-3	3-4	8 42.85	66 14 42	66 11 10	1.0	+ 1.98	— 1.10	— 6.34	+ 8.00	
485	6	3 & 3-4	3 & 3-4	8 9.79	67 2 31	66 57 23	0.5	— 2.23	+ 0.41	— 8.75	+ 5.22	-
513	27	2-3	2-3	6 31.25	243 40 14	243 47 53	1.0	+ 0.70	— 2.00	+ 4.74	+ 8.17	-
547	Dec. 14	2	3	7 0.98	244 53 9	244 52 18	1.0	— 1.78	+ 0.91	+ 3.48	+ 8.23	

OBSERV

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coeffic Δp_1	Coeffic Δq_1	Coeffic Δp_4	Coeffic Δq_4	
	1891			h m	° ′ ″	° ′ ″						
85	Aug. 27	3-4	3	7 40.36	63 28 5	63 27 40	0.25	— 1.20	+ 2.25	+ 6.00	+ 9.61	+
360	Oct. 6	2-3	3	5 53.87	232 50 24	233 13 36	1.0	— 1.89	+ 1.47	+ 10.30	+ 3.31	-
403	14	2-3	3	9 16.06	262 15 13	261 30 3	1.0	+ 2.33	— 0.06	— 9.94	— 2.61	+
425	16	2-3	3	9 24.62	64 33 2	64 33 36	1.0	— 1.66	— 1.75	+ 5.65	+ 9.11	+
435	17	2-3	3-4	8 46.47	64 58 58	64 57 18	1.0	+ 1.00	+ 2.18	+ 2.03	+ 10.54	+
449	19	2	3	9 35.84	66 9 5	66 4 55	1.0	— 1.17	+ 2.09	— 5.77	+ 9.05	+
529	Dec. 8	2	3	6 32.00	66 6 44	66 2 24	1.0	— 2.03	— 0.24	— 4.36	+ 8.06	-

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OSITION-ANGLES.

LL

	Δk_1	Δh_4	Δk_4	y_4	$\Delta\varepsilon_1$	$\Delta\varepsilon_4$	O—C	(O—C) × s sin 1″	Differential aberration	Residuals	rot. Nr
							′ ″	″	″	″	
04	— 1.24	— 8.05	+ 4.17	+ 0.14	+ 1.46	+ 9.04	— 36 48	— 0.247	— 0.011	— 0.236	2
01	— 0.08	— 0.14	+ 0.39	+ 0.01	+ 0.08	+ 0.41	+ 0 31	+ 0.109	— 0.001	+ 0.051	17
01	— 0.06	— 0.06	+ 0.40	+ 0.02	— 0.04	+ 0.40	+ 0 7	+ 0.014	0.000	+ 0.007	37
05	— 0.01	+ 0.35	+ 0.09	— 0.01	— 0.02	+ 0.36	— 6 44	— 0.861	0.000	— 0.145	59
73	+ 0.74	+ 6.18	— 2.38	— 0.06	+ 0.82	+ 6.59	— 24 47	— 0.230	+ 0.008	+ 0.020	82
04	+ 0.04	— 0.08	— 0.28	0.00	+ 0.05	+ 0.29	— 1 45	— 0.369	0.000	+ 0.050	92
06	+ 0.01	— 0.33	— 0.18	+ 0.01	+ 0.04	+ 0.37	— 4 56	— 0.784	0.000	— 0.042	124
04	+ 0.02	— 0.30	— 0.13	+ 0.01	— 0.01	+ 0.33	— 6 16	— 0.948	0.000	— 0.181	130
02	— 0.06	— 0.24	+ 0.50	— 0.02	— 0.01	+ 0.55	+ 1 48	+ 0.170	— 0.001	— 0.044	150
02	— 0.04	+ 0.08	+ 0.28	— 0.01	— 0.01	+ 0.29	— 0 19	— 0.056	0.000	+ 0.269	161
4	+ 0.06	+ 1.11	— 0.12	+ 0.02	+ 0.07	+ 1.11	— 11 50	— 0.592	+ 0.001	+ 0.018	195
5	+ 0.06	+ 0.33	— 0.53	+ 0.02	+ 0.03	+ 0.62	+ 1 51	+ 0.162	+ 0.001	— 0.119	213
3	+ 0.04	— 0.01	— 0.28	+ 0.01	+ 0.03	+ 0.28	— 1 54	— 0.350	0.000	— 0.133	226
9	+ 0.04	— 0.60	+ 0.07	— 0.01	+ 0.07	+ 0.60	— 8 50	— 0.768	— 0.001	+ 0.049	258
5	— 0.09	— 0.63	+ 0.52	0.00	+ 0.04	+ 0.81	+ 4 39	+ 0.299	— 0.001	— 0.081	266
0	— 0.06	— 0.55	+ 0.66	— 0.01	+ 0.08	+ 0.85	+ 6 48	+ 0.437	— 0.001	+ 0.048	295
1	— 0.04	— 0.02	+ 0.29	0.00	+ 0.01	+ 0.29	+ 0 39	+ 0.105	0.000	+ 0.172	309
6	+ 0.03	— 0.41	— 0.13	+ 0.01	— 0.03	+ 0.43	— 6 37	— 0.683	0.000	— 0.001	344
1	— 0.07	— 0.03	+ 0.29	+ 0.01	+ 0.06	+ 0.29	+ 0 24	+ 0.076	0.000	+ 0.186	376
1	— 0.03	+ 0.45	+ 0.06	— 0.01	+ 0.11	+ 0.45	— 6 45	— 0.778	+ 0.001	+ 0.001	397
4	+ 0.04	— 0.12	— 0.17	0.00	— 0.05	+ 0.21	— 3 32	— 0.475	0.000	— 0.051	473
7	+ 0.02	— 0.24	— 0.13	— 0.01	+ 0.07	+ 0.27	— 5 8	— 0.807	0.000	— 0.058	485
2	— 0.07	— 0.23	+ 0.34	+ 0.01	— 0.06	+ 0.41	+ 7 39	+ 0.443	— 0.001	+ 0.187	513
:	— 0.03	— 0.06	+ 0.21	— 0.01	+ 0.03	+ 0.21	— 0 51	— 0.107	0.000	— 0.129	547

LAY

	Δk_1	Δh_4	Δk_4	y_4	$\Delta\varepsilon_1$	$\Delta\varepsilon_4$	O—C	(O—C) × s sin 1″	Differential aberration	Residuals	rot. Nr
							′ ″	″	″	″	
	+ 0.09	+ 0.22	— 0.41	0.00	+ 0.08	+ 0.46	— 0 25	— 0.064	+ 0.001	— 0.211	85
	— 0.08	— 2.25	+ 1.02	+ 0.04	— 0.20	+ 2.46	+ 23 12	+ 0.387	— 0.004	+ 0.060	360
	+ 0.10	+ 2.88	— 0.84	— 0.05	— 0.65	+ 3.00	— 45 10	— 0.477	+ 0.004	— 0.003	403
	+ 0.07	+ 0.17	— 0.31	0.00	— 0.07	+ 0.35	+ 0 34	+ 0.049	0.000	— 0.163	425
	+ 0.07	+ 0.07	— 0.29	0.00	+ 0.07	+ 0.30	— 1 40	— 0.292	0.000	— 0.263	435
	+ 0.05	— 0.12	— 0.20	0.00	+ 0.06	+ 0.23	— 4 10	— 0.902	0.000	— 0.299	449
	+ 0.02	— 0.10	— 0.15	0.00	+ 0.03	+ 0.18	— 4 20	— 0.707	0.000	— 0.234	529

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TABLE X. SATELLITES II and III

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficien μ	z_G	x_2	Δh_2	Δk_2	y_2	z_2
	1891			h m	"	"									
12	Aug. 17	2	2	10 38.54	87.822	88.374	c	1.0	— 0.88	— 1.13	+ 0.51	— 2.48	+ 0.46	— 0.20	— 2
22	19	2-3	3	9 55.04	331.516	331.985	c	1.0	— 3.32	— 0.30	+ 0.47	— 3.06	+ 0.66	+ 0.01	+ 0
36	20	2	3	9 16.61	325.749	325.346	c	1.0	— 3.26	— 0.31	+ 1.33	+ 1.89	— 2.11	+ 0.04	+ 2
52	24	2-3	2-3	7 56.92	13.498	13.628	d	0.5	— 0.14	0.00	— 0.12	— 1.60	+ 0.62	— 0.08	— 2
74	25	3	3	8 15.86	436.333	436.649	c	0.5	— 4.36	— 0.23	— 1.44	— 3.13	+ 2.42	— 0.39	— 0
105	29	2-3	2-3	9 5.54	122.335	122.139	c	1.0	— 1.22	— 0.82	+ 1.41	+ 1.69	— 1.85	— 0.19	— 0
109	31	3-4	3-4	7 20.52	30.726	30.613	ψ	0.25	— 0.31	0.00	+ 0.78	+ 1.66	— 1.15	+ 0.03	+ 2
140	Sept. 1	—		8 45.12	408.052	408.184	c	1.0	— 4.08	— 0.25	— 1.52	— 3.40	+ 2.22	— 0.40	— 1
154	5	2	2-3	8 25.07	167.391	167.141	c	1.0	— 1.67	— 0.60	+ 1.88	+ 1.96	— 2.18	— 0.25	+ 0
172	7	1-2	2-3	7 59.32	56.288	56.121	ψ	1.0	— 0.56	0.00	+ 1.26	+ 1.58	— 1.46	— 0.01	+ 3
175	8	—		6 39.32	322.457	322.573	c	1.0	— 3.22	— 0.31	— 1.18	— 3.74	+ 1.58	— 0.40	+ 0
186	10	2	2-3	6 32.13	33.544	33.093	ψ	1.0	— 0.36	0.00	+ 1.63	+ 2.87	— 1.86	+ 0.08	+ 1
202	13	2	3	6 44.79	48.208	47.920	c	1.0	— 0.48	— 2.09	+ 0.42	+ 3.23	— 0.55	— 0.03	+ 0
217	14	3	3 & 3-4	6 13.02	92.936	92.800	ψ	0.5	— 0.93	0.00	+ 1.84	+ 1.91	— 1.89	— 0.07	+ 3
253	19	2	3	6 26.26	266.558	266.398	c	1.0	— 2.67	— 0.38	+ 2.47	+ 2.92	— 2.36	— 0.35	+ 1
279	20	1-2	2	6 57.45	57.225	57.032	ψ	1.0	— 0.57	0.00	+ 0.52	+ 2.96	— 0.38	— 0.04	+ 1
284	21	1	1-2	6 8.78	113.420	113.215	c	1.0	— 1.13	— 0.88	+ 2.21	+ 2.18	— 2.09	— 0.10	+ 3
304	22	2	2-3	6 37.46	213.344	213.547	c	1.0	— 2.13	— 0.47	— 1.06	— 3.63	+ 0.79	— 0.36	— 3
321	23	2	2-3	6 28.14	560.668	560.974	c	1.0	— 5.61	— 0.18	— 1.65	— 1.71	+ 1.51	— 0.16	— 0
343	Oct. 4	3	3	5 16.85	82.741	82.327	ψ	0.5	— 0.83	0.00	+ 0.93	+ 2.07	— 0.47	— 0.07	+ 1
379	9	2	2-3	10 8.38	378.951	378.338	c	0.0	— 3.79	— 0.26	+ 1.18	+ 1.84	— 0.67	— 0.06	+ 4
387	13	1-2	1-2	8 58.22	85.719	85.934	c	1.0	— 0.86	— 1.16	— 1.23	— 3.00	+ 0.22	— 0.27	— 4
478	Nov. 6	2	2-3	7 16.97	75.713	75.152	c	1.0	— 0.76	— 1.33	+ 2.86	+ 2.28	— 1.85	+ 0.17	+ 4
490	8	2	2-3	7 17.91	147.869	147.790	c	1.0	— 1.48	— 0.68	+ 2.29	+ 1.71	— 1.56	— 0.16	+ 2
505	22	2	2-3	6 57.35	177.521	177.678	c	1.0	— 1.78	— 0.56	+ 2.87	+ 2.16	— 1.89	— 0.20	+ 3

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficien μ	z_F	x_2	Δh_2	Δk_2	y_2	z_2
	1891			h m	"	"									
243	Sept. 18	2	3	7 8.76	460.095	459.494	c	1.0	— 4.60	— 0.22	+ 0.43	+ 2.30	— 0.38	— 0.16	+ 3
410	Oct. 14	3	3-4	9 55.66	535.179	535.635	c	0.5	— 5.35	— 0.19	— 2.37	— 1.90	+ 1.82	— 0.20	— 1
418	15	3-4	3-4 & 4	9 13.34	280.516	281.163	c	0.25	— 2.81	— 0.36	— 1.98	— 3.50	+ 0.75	— 0.11	—
422	16	2-3	3	9 2.37	313.284	312.528	c	1.0	— 3.13	— 0.32	+ 1.66	+ 1.70	— 1.11	+ 0.02	+ 1
444	19	2	3	8 54.56	161.000	160.625	c	1.0	— 1.61	— 0.62	+ 3.00	+ 3.00	— 1.94	— 0.16	+ 3
458	23	2	3	8 18.53	239.270	238.511	c	1.0	— 2.39	— 0.42	+ 2.13	+ 1.78	— 1.48	+ 0.08	+ 2
538	Dec. 8	3	3-4	7 30.53	238.457	238.078	c	0.5	— 2.38	— 0.42	+ 2.55	+ 1.94	— 1.76	— 0.19	+ 4

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Δh_3	Δk_3	y_3	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	Δp_2	Δq_2	Δp_3	Δq_3	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
2.98	— 3.44	+ 0.71	+ 1.77	+ 3.75	— 0.31	+ 0.21	— 0.06	— 0.60	+ 0.552	+ 0.002	+ 0.297	12
5.65	— 1.73	+ 0.56	— 2.78	— 5.72	— 0.14	+ 0.04	— 0.24	— 0.01	+ 0.469	+ 0.005	— 0.073	22
4.64	+ 3.93	— 0.72	+ 2.31	+ 5.96	— 0.02	— 0.19	— 0.19	— 0.24	— 0.403	— 0.008	+ 0.119	36
3.03	— 3.11	+ 0.30	+ 0.36	+ 3.96	— 1.28	+ 1.87	— 1.21	— 3.39	+ 0.130	+ 0.006	— 0.390	52
2.89	— 1.45	+ 0.38	+ 3.92	— 0.25	— 0.04	— 0.04	+ 0.05	— 0.08	+ 0.316	— 0.010	+ 0.075	74
3.95	+ 0.74	— 0.54	— 1.69	— 2.77	+ 0.02	— 0.27	— 0.34	+ 0.25	— 0.196	— 0.001	— 0.035	105
4.04	+ 3.83	— 0.32	+ 0.49	— 5.30	— 0.34	+ 0.87	— 0.71	— 1.30	— 0.113	— 0.009	— 0.008	109
2.78	— 1.71	+ 0.17	+ 4.05	+ 0.41	— 0.06	— 0.05	+ 0.05	— 0.11	+ 0.132	— 0.010	— 0.101	140
3.54	+ 0.83	+ 0.22	— 2.44	— 1.92	— 0.03	— 0.18	— 0.20	+ 0.20	— 0.250	+ 0.003	— 0.159	154
4.50	+ 3.81	+ 0.80	+ 1.06	— 5.71	— 0.12	+ 0.47	— 0.45	— 0.63	— 0.167	— 0.009	— 0.037	172
2.71	— 2.28	— 0.48	+ 4.05	+ 1.66	— 0.11	— 0.04	+ 0.04	— 0.18	+ 0.116	— 0.009	— 0.083	175
5.15	+ 2.32	+ 0.66	+ 3.39	+ 5.64	— 1.49	— 1.32	— 2.96	— 1.24	— 0.468	— 0.004	+ 0.109	186
5.55	+ 1.17	+ 0.47	+ 3.00	— 5.39	+ 0.42	— 0.11	— 0.69	+ 0.07	— 0.288	— 0.003	— 0.057	202
5.12	+ 3.55	+ 0.72	+ 2.05	— 6.16	+ 0.02	+ 0.25	— 0.29	— 0.28	— 0.136	+ 0.003	+ 0.024	217
2.88	+ 1.44	+ 0.38	— 3.65	+ 0.08	— 0.05	— 0.06	— 0.05	+ 0.11	— 0.160	+ 0.009	— 0.071	253
5.28	+ 0.83	+ 0.78	+ 2.56	— 4.95	+ 0.32	— 0.14	— 0.56	+ 0.13	— 0.193	— 0.004	— 0.059	279
5.46	+ 3.20	+ 0.66	+ 2.60	— 6.30	+ 0.05	+ 0.18	— 0.24	— 0.17	— 0.205	— 0.009	— 0.022	284
2.99	— 2.90	— 0.29	+ 3.59	+ 3.07	— 0.18	+ 0.01	— 0.02	— 0.29	+ 0.203	— 0.005	— 0.045	304
3.75	— 0.56	— 0.53	+ 1.24	— 2.34	0.00	0.00	0.00	0.00	+ 0.306	+ 0.002	— 0.012	321
4.31	+ 0.37	— 0.30	+ 0.92	— 3.44	+ 0.14	— 0.18	— 0.32	+ 0.20	— 0.414	— 0.003	— 0.242	343
4.68	+ 3.38	— 0.67	— 0.20	+ 5.62	+ 0.05	— 0.09	— 0.11	— 0.12	— 0.613	— 0.011	[— 0.069]	379
3.62	— 3.21	+ 0.38	+ 2.65	+ 4.32	— 0.38	+ 0.16	— 0.24	— 0.60	+ 0.215	+ 0.001	— 0.085	387
5.37	+ 1.69	+ 0.64	+ 2.70	+ 5.60	— 0.25	— 0.50	— 0.86	— 0.19	— 0.561	— 0.006	+ 0.017	478
2.63	+ 1.11	+ 0.36	— 1.70	0.00	— 0.01	— 0.06	— 0.04	+ 0.10	— 0.079	+ 0.002	+ 0.075	490
2.41	+ 1.88	— 0.41	— 2.67	+ 1.58	— 0.01	— 0.02	0.00	+ 0.04	+ 0.157	+ 0.004	+ 0.375	505

NLAY

h_3	Δk_3	y_3	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	Δp_2	Δq_2	Δp_3	Δq_3	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
3.88	+ 3.67	+ 0.40	— 1.34	+ 4.91	+ 0.05	— 0.05	— 0.04	— 0.10	— 0.601	— 0.007	— 0.132	243
3.02	— 0.88	+ 0.44	+ 2.07	— 0.78	0.00	— 0.02	+ 0.02	— 0.03	+ 0.456	— 0.003	+ 0.207	410
5.32	— 0.75	+ 0.79	— 3.51	— 5.09	— 0.15	0.00	— 0.23	+ 0.04	+ 0.647	+ 0.001	+ 0.221	418
5.09	+ 3.07	— 0.62	+ 0.76	+ 5.88	+ 0.03	— 0.14	— 0.18	— 0.15	— 0.756	— 0.012	— 0.099	422
5.68	+ 1.70	— 0.73	+ 3.50	— 5.89	+ 0.06	+ 0.06	— 0.12	— 0.02	— 0.375	— 0.004	— 0.003	444
5.35	+ 2.66	+ 0.03	+ 1.56	+ 5.96	0.00	— 0.18	— 0.26	— 0.15	— 0.759	— 0.010	— 0.089	458
4.73	+ 1.21	— 0.63	+ 2.38	— 4.79	+ 0.01	+ 0.03	— 0.05	0.00	— 0.379	— 0.004	— 0.108	538

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TABLE XI. SATELLITES II and III

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficients Δp_2	Δq_2	Δp_3	Δq_3	x_2	Δ
	1891			h m	° ′ ″	° ′ ″							
11	Aug. 17	2	2	10 35.41	59 36 59	59 17 11	1.0	+ 3.33	— 2.25	+ 0.69	+ 6.33	— 0.05	—
21	19	2-3	3	9 55.25	67 12 43	67 14 37	1.0	— 3.84	+ 1.08	— 6.42	— 0.31	— 0.10	+
35	20	2	3	9 14.71	242 20 49	242 20 58	1.0	+ 0.39	+ 3.99	+ 3.99	+ 5.04	+ 0.15	+
51	24	2-3	2-3	7 56.92	31 6 40	28 56 45	1.0	+ 1.90	— 2.77	+ 1.80	+ 5.02	— 0.20	—
73	25	3	3	8 15.75	64 13 39	64 13 38	0.5	+ 2.86	+ 2.91	— 3.35	+ 5.52	— 0.07	—
104	29	2-3	2-3	9 5.15	248 57 41	248 45 48	1.0	+ 0.29	— 4.04	— 5.22	+ 3.78	0.00	—
108	31	3-4	3-4	7 20.61	258 30 2	258 44 31	0.25	— 1.45	+ 3.68	— 3.02	— 5.52	— 0.09	—
139	Sept. 1	—		8 44.54	64 6 41	64 8 26	1.0	+ 3.26	+ 2.48	— 2.69	+ 5.89	— 0.07	—
153	5	2	2-3	8 24.86	247 47 53	247 31 49	1.0	— 0.65	— 4.02	— 4.57	+ 4.58	— 0.01	—
171	7	1-2	2-3	7 59.48	252 10 8	252 44 53	1.0	— 0.97	+ 3.92	— 3.70	— 5.25	— 0.05	—
174	8	—		6 39.18	63 40 28	63 38 9	1.0	+ 3.90	+ 1.29	— 1.39	+ 6.33	— 0.07	—
185	10	2	2-3	6 32.83	215 53 24	216 0 31	1.0	+ 2.62	+ 2.32	+ 5.21	+ 2.17	+ 0.97	+
201	13	2	3	6 44.49	251 31 8	251 44 6	1.0	+ 3.86	— 1.00	— 6.38	+ 0.67	+ 0.01	—
216	14	3	3 & 3-4	6 12.74	248 56 32	249 10 25	0.5	+ 0.28	+ 4.03	— 4.68	— 4.41	— 0.04	—
252	19	2	3	6 25.82	246 30 55	246 19 12	1.0	— 2.50	— 3.19	— 2.74	+ 5.82	0.00	—
278	20	1-2	2	6 57.54	250 34 2	250 31 54	1.0	+ 3.63	— 1.62	— 6.22	+ 1.48	+ 0.01	—
283	21	1	1-2	6 8.45	248 5 56	247 58 17	0.0	+ 1.02	+ 3.88	— 5.22	— 3.70	— 0.04	—
303	22	2	2-3	6 37.15	62 53 35	62 53 18	1.0	+ 4.05	— 0.17	+ 0.36	+ 6.39	— 0.08	—
320	23	2	2-3	6 27.81	65 28 7	65 28 55	1.0	— 0.51	+ 3.98	— 4.67	+ 4.39	— 0.08	+
342	Oct. 4	3	3	5 16.50	249 4 23	248 45 52	1.0	+ 2.40	— 3.08	— 5.29	+ 3.36	0.00	—
378	9	2	2-3	10 9.16	244 11 44	244 9 49	1.0	— 1.76	+ 3.50	+ 4.06	+ 4.72	+ 0.09	
386	13	1-2	1-2	8 58.75	59 43 27	59 31 55	1.0	+ 3.56	— 1.55	+ 2.26	+ 5.68	— 0.15	—
467	Nov. 5	2-3	3-4	7 43.63	66 21 15		0.0	— 2.85	+ 2.16	— 4.61	+ 3.46	+ 0.01	+
477	6	2	2-3	7 17.13	236 59 29	237 11 0	0.0	+ 1.60	+ 3.18	+ 5.56	+ 1.22	+ 0.47	+
489	8	2	2-3	7 17.86	246 50 31	246 31 55	1.0	— 0.36	— 3.54	— 2.21	+ 5.26	— 0.02	—
504	22	2	2-3	6 57.24	246 9 10	245 56 10	1.0	— 1.60	— 3.01	— 0.62	+ 5.42	— 0.01	—

OBSERV

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficients Δp_2	Δq_2	Δp_3	Δq_3	x_2	Δ
	1891			h m	° ′ ″	° ′ ″							
242	Sept. 18	2	3	7 8.63	244 25 45	244 22 40	1.0	— 2.88	+ 2.87	+ 2.58	+ 5.91	+ 0.08	—
409	Oct. 14	3	3-4	9 55.82	65 22 30	65 22 29	0.5	+ 0.61	+ 3.80	— 3.14	+ 5.28	— 0.06	+
417	15	3-4	3-4 & 4	9 14.69	67 57 29	67 52 28	0.25	+ 3.80	— 0.04	— 6.03	+ 1.13	+ 0.07	+
421	16	2-3	3	9 1.86	243 43 16	243 41 55	1.0	— 0.78	+ 3.77	+ 4.74	+ 3.88	+ 0.11	+
443	19	2	3	8 54.05	246 58 30	247 9 13	1.0	+ 2.66	+ 2.70	— 5.95	— 1.05	— 0.04	—
457	23	2	3	8 17.98	242 59 23	242 59 22	1.0	+ 0.11	+ 3.76	+ 5.20	+ 3.00	+ 0.15	+
537	Dec. 8	3	3-4	7 30.79	246 10 4	246 16 19	0.5	+ 1.22	+ 3.02	— 5.16	— 0.26	— 0.03	—

283 Rejected; included in first approximation. $n = (O - C)s \sin 1''$ from Cape solution = + 0″.628 (see page 58).
467 „ „ „ „ „ „ „ „ „ „ „ „ = — 0.278 („ „ 58).

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OSITION-ANGLES.

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x_2	y_2	x_3	Δh_3	Δk_3	y_3	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	O—C	(O—C)× s sin 1″	Differential aberration	Residuals	rot. Nr.
								′ ″	″	″	″	
12	— 0.01	— 0.35	+ 0.24	— 0.48	+ 0.09	+ 0.06	+ 0.50	— 19 48	— 0.506	+ 0.001	— 0.148	11
03	0.00	— 0.12	— 0.31	— 0.01	— 0.02	+ 0.20	+ 0.31	+ 1 54	+ 0.184	— 0.001	+ 0.356	21
17	+ 0.01	+ 0.21	— 0.19	+ 0.29	— 0.04	+ 0.18	+ 0.34	+ 0 9	+ 0.014	— 0.001	+ 0.049	35
52	— 0.05	— 1.58	+ 2.01	— 2.26	+ 0.19	+ 0.11	+ 2.81	— 129 55	— 0.511	+ 0.004	— 0.203	51
08	— 0.01	— 0.19	— 0.06	— 0.18	— 0.02	+ 0.06	+ 0.19	— 0 1	— 0.002	0.000	— 0.083	73
04	+ 0.01	+ 0.17	+ 0.39	+ 0.08	+ 0.06	+ 0.03	+ 0.36	— 11 53	— 0.421	0.000	+ 0.143	104
18	— 0.02	— 0.56	+ 0.98	— 0.78	+ 0.09	— 0.01	+ 1.16	+ 14 29	+ 0.129	+ 0.002	— 0.523	108
08	— 0.01	— 0.20	— 0.04	— 0.20	— 0.02	+ 0.06	+ 0.19	+ 1 45	+ 0.208	0.000	+ 0.159	139
03	+ 0.01	+ 0.15	+ 0.27	+ 0.11	+ 0.01	+ 0.04	+ 0.27	— 16 4	— 0.781	0.000	— 0.089	153
08	— 0.01	— 0.28	+ 0.57	— 0.35	— 0.08	— 0.02	+ 0.61	+ 34 45	+ 0.569	+ 0.001	— 0.087	171
08	— 0.01	— 0.16	+ 0.01	— 0.23	— 0.04	+ 0.07	+ 0.22	— 2 19	— 0.217	0.000	— 0.179	174
10	+ 0.05	+ 1.18	— 2.86	+ 1.41	+ 0.38	+ 1.92	+ 3.18	+ 7 7	+ 0.070	— 0.012	+ 0.098	185
01	0.00	— 0.01	+ 0.70	— 0.04	— 0.05	— 0.41	+ 0.69	+ 12 58	+ 0.182	0.000	— 0.004	201
04	0.00	— 0.12	+ 0.35	— 0.13	— 0.04	— 0.06	+ 0.34	+ 13 53	+ 0.376	+ 0.001	— 0.304	216
00	+ 0.01	+ 0.13	+ 0.14	+ 0.13	— 0.03	+ 0.05	+ 0.18	— 11 43	— 0.907	0.000	— 0.053	252
02	0.00	— 0.01	+ 0.58	+ 0.02	— 0.09	— 0.32	+ 0.57	— 2 8	— 0.036	0.000	— 0.086	278
03	0.00	— 0.08	+ 0.29	— 0.06	— 0.04	— 0.06	+ 0.27	— 7 39	— 0.252	+ 0.001	[— 0.932]	283
08	— 0.01	— 0.28	+ 0.10	— 0.28	0.00	+ 0.09	+ 0.29	— 0 17	— 0.017	0.000	+ 0.132	303
08	0.00	— 0.13	— 0.10	— 0.14	+ 0.02	+ 0.08	+ 0.17	+ 0 48	+ 0.131	0.000	+ 0.129	320
04	0.00	0.00	+ 0.36	+ 0.09	+ 0.01	— 0.15	+ 0.35	— 18 31	— 0.446	0.000	— 0.150	342
09	+ 0.01	+ 0.20	— 0.10	+ 0.18	— 0.02	+ 0.08	+ 0.20	— 1 55	— 0.212	0.000	— 0.021	378
07	— 0.02	— 0.55	+ 0.36	— 0.46	+ 0.03	+ 0.20	+ 0.56	— 11 32	— 0.288	0.000	— 0.049	386
05	0.00	— 0.01	— 0.11	— 0.10	+ 0.01	+ 0.09	+ 0.15					467
33	+ 0.03	+ 0.73	— 0.79	+ 0.32	+ 0.09	+ 0.45	+ 0.84	+ 11 31	+ 0.254	— 0.001	[+ 0.543]	477
03	0.00	+ 0.01	+ 0.10	+ 0.10	— 0.02	— 0.02	+ 0.13	— 18 36	— 0.800	0.000	— 0.116	489
02	0.00	+ 0.02	+ 0.05	+ 0.10	— 0.01	0.00	+ 0.11	— 13 0	— 0.672	0.000	+ 0.077	504

NLAY

x_2	y_2	x_3	Δh_3	Δk_3	y_3	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	O—C	(O—C)× s sin 1″	Differential aberration	Residuals	rot. Nr.
								′ ″	″	″	″	
08	+ 0.01	+ 0.19	— 0.04	+ 0.19	0.00	+ 0.08	+ 0.19	— 3 5	— 0.413	0.000	— 0.014	242
07	0.00	— 0.11	— 0.06	— 0.14	— 0.01	+ 0.07	+ 0.15	— 0 1	— 0.003	0.000	— 0.006	409
00	0.00	+ 0.06	— 0.29	— 0.05	— 0.05	+ 0.19	+ 0.29	— 5 1	— 0.409	0.000	— 0.595	417
11	+ 0.01	+ 0.25	— 0.17	+ 0.20	— 0.02	+ 0.11	+ 0.26	— 1 21	— 0.123	— 0.001	— 0.077	421
01	0.00	— 0.05	+ 0.18	+ 0.03	+ 0.03	— 0.07	+ 0.17	+ 10 43	+ 0.502	0.000	— 0.004	443
14	+ 0.01	+ 0.28	— 0.23	+ 0.20	+ 0.01	+ 0.14	+ 0.30	— 0 1	— 0.001	— 0.001	— 0.076	457
02	0.00	— 0.07	+ 0.10	+ 0.04	+ 0.02	0.00	+ 0.10	+ 6 15	+ 0.365	0.000	— 0.010	537

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TABLE XII. SATELLITES II and IV

OBSERVER

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficients μ	z_G	x_2	Δh_2	Δk_2	y_2
	1891			h m	"	"								
6	Aug. 17	2	2	9 45.73	131.921	131.038	c	1.0	— 1.32	— 0.76	— 0.44	+ 2.34	— 0.49	+ 0.1
24	19	2-3	3	10 15.69	225.416	225.317	c	1.0	— 2.25	— 0.44	— 0.47	+ 3.11	— 0.68	— 0.0
30	20	2-3	3	8 18.85	715.976	715.802	c	1.0	— 7.16	— 0.14	+ 1.39	+ 2.00	— 2.21	+ 0.2
58	24	2-3	2-3	8 41.80	603.546	603.311	c	1.0	— 6.04	— 0.17	+ 0.12	+ 1.95	— 0.72	+ 0.2
78	25	2-3	2-3	8 39.44	81.262	80.581	ψ	1.0	— 0.81	0.00	+ 1.44	+ 2.94	— 2.36	+ 0.3
95	29	2-3	3	7 59.60	829.026	829.140	c	1.0	— 8.29	— 0.12	— 1.53	— 1.76	+ 2.00	+ 0.2
115	31	3-4	3-4	8 7.77	361.091	361.622	c	0.25	— 3.61	— 0.28	— 0.66	— 1.75	+ 1.05	— 0.2
138	Sept. 1	—		8 34.04	480.038	480.813	c	1.0	— 4.80	— 0.21	— 1.50	— 3.42	+ 2.20	— 0.4
158	5	2	2-3	8 46.40	262.823	262.403	c	1.0	— 2.63	— 0.38	+ 1.85	+ 1.92	— 2.14	— 0.2
168	7	1-2	2-3	7 36.51	873.446	873.501	c	1.0	— 8.73	— 0.11	+ 1.21	+ 1.72	— 1.42	— 0.0
177	8	—		6 53.16	649.371	649.168	c	1.0	— 6.49	— 0.15	+ 1.22	+ 3.74	— 1.62	+ 0.4
192	10	2	2-3	7 13.82	370.199	370.161	c	1.0	— 3.70	— 0.27	+ 1.96	+ 3.20	— 2.22	+ 0.1
212	13	2	3	7 55.56	539.536	539.591	c	1.0	— 5.40	— 0.19	— 0.51	— 3.40	+ 0.65	+ 0.2
223	14	3-4 & 4	3-4 & 4	6 52.44	354.356	354.465	c	0.25	— 3.54	— 0.28	— 1.75	— 1.87	+ 1.81	+ 0.2
230		1-2	1-2	7 41.41	352.408	352.525	c	1.0	— 3.52	— 0.28	— 1.65	— 1.82	+ 1.70	+ 0.2
261	19	2-3	3	7 19.18	267.086	267.659	c	1.0	— 2.67	— 0.37	— 2.48	— 2.78	+ 2.38	+ 0.3
277	20	1-2	2	6 48.23	91.063	91.505	c	1.0	— 0.91	— 1.09	— 0.51	— 2.89	+ 0.37	+ 0.2
288	21	1	1-2	6 29.20	494.399	494.086	c	1.0	— 4.94	— 0.20	+ 2.18	+ 2.14	— 2.05	— 0.2
312	22	2	2-3	7 20.10	603.831	603.609	c	1.0	— 6.04	— 0.17	+ 1.16	+ 3.67	— 0.89	+ 0.3
323	23	2	2-3	6 41.92	394.724	394.600	c	1.0	— 3.95	— 0.25	+ 1.61	+ 1.71	— 1.47	+ 0.1
353	Oct. 4	3	3	6 20.22	692.816	693.504	c	0.5	— 6.93	— 0.14	— 0.87	— 2.20	+ 0.38	+ 0.0
375	9	2-3	3	9 48.81	778.216	777.967	c	1.0	— 7.78	— 0.13	+ 1.21	+ 1.82	— 0.71	— 0.0
396	13	2-3	2-3	9 48.19	462.734	462.561	c	1.0	— 4.63	— 0.22	+ 1.33	+ 3.11	— 0.28	+ 0.2
472	Nov. 5	2-3	3-4	8 25.28	736.243	736.997	c	1.0	— 7.36	— 0.14	— 1.61	— 2.46	+ 0.15	— 0.0
492	8	2	2-3	7 30.13	297.679	298.272	c	1.0	— 2.98	— 0.34	— 2.27	— 1.70	+ 1.54	+ 0.3
511	22	3	3-4	7 43.10	648.105	648.316	c	0.5	— 6.48	— 0.15	— 2.78	— 2.07	+ 1.86	+ 0.1
544	Dec. 14	2	3	6 32.11	163.404	162.972	c	1.0	— 1.63	— 0.61	+ 1.72	+ 1.74	— 0.34	— 0.1

OBSERVER

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficients μ	z_F	x_2	Δh_2	Δk_2	y_2
	1891			h m	"	"								
88	Aug. 27	3-4	3	8 20.80	171.313	170.694	c	0.25	— 1.71	— 0.58	— 1.68	— 2.31	+ 2.35	— 0.0
359	Oct. 6	2-3	3	5 37.77	106.300	105.185	c	1.0	— 1.06	— 0.95	+ 0.93	+ 2.90	— 0.21	+ 0.2
402	14	2-3	3	9 0.88	83.829	83.743	ψ	1.0	— 0.84	0.00	— 2.43	— 1.96	+ 1.85	— 0.2
416	15	3	3	8 58.92	224.156	224.175	c	0.5	— 2.24	— 0.45	— 1.95	— 3.49	+ 0.72	— 0.1
420	16	2-3	3	8 51.44	131.776	131.505	c	1.0	— 1.32	— 0.76	— 1.68	— 1.70	+ 1.13	+ 0.2
454	19	2	3	10 1.06	507.832	508.219	c	1.0	— 5.08	— 0.20	— 2.99	— 2.86	+ 1.98	+ 0.1
456	23	2	3	8 6.19	232.965	232.245	c	1.0	— 2.33	— 0.43	+ 2.16	+ 1.79	— 1.51	+ 0.2
536	Dec. 8	3	3-4	7 19.33	397.494	397.950	c	0.5	— 3.98	— 0.25	— 2.57	— 1.96	+ 1.77	+ 0.1

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ISTANCES

ILL

h_4	Δk_4	y_4	$\Delta\varepsilon_2$	$\Delta\varepsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
).96	+ 5.07	+ 0.16	— 1.53	+ 11.16	+ 0.39	— 0.30	— 1.28	— 0.54	— 0.883	— 0.015	— 0.148	6
).63	+ 7.03	+ 0.41	+ 2.86	+ 9.06	— 0.26	+ 0.07	— 0.31	— 0.71	— 0.099	— 0.007	+ 0.230	24
;.06	+ 5.96	+ 0.68	+ 2.54	+ 6.23	— 0.01	— 0.07	— 0.01	— 0.20	— 0.174	— 0.002	— 0.102	30
).48	+ 2.21	+ 0.25	— 0.58	— 9.16	— 0.06	+ 0.07	— 0.26	+ 0.03	— 0.235	— 0.012	— 0.023	58
).80	+ 3.98	+ 0.09	— 3.72	— 10.50	— 0.78	— 0.85	— 3.06	— 0.86	— 0.681	— 0.007	— 0.198	78
4.71	— 4.35	— 0.46	+ 1.98	+ 3.40	0.00	— 0.04	+ 0.02	— 0.10	+ 0.114	— 0.001	+ 0.006	95
7.04	— 1.23	— 0.15	— 0.25	— 4.96	+ 0.05	— 0.11	— 0.29	+ 0.21	+ 0.531	+ 0.003	+ 0.062	115
).16	— 1.69	— 0.01	+ 4.06	— 8.47	+ 0.05	+ 0.04	— 0.17	+ 0.04	+ 0.775	0.000	+ 0.098	138
;.27	+ 6.63	— 0.11	— 2.35	+ 8.21	+ 0.01	+ 0.08	— 0.07	— 0.22	— 0.420	— 0.005	— 0.222	158
4.91	+ 2.96	— 0.39	+ 1.06	+ 0.67	0.00	— 0.01	+ 0.01	— 0.02	+ 0.055	+ 0.001	+ 0.081	168
;.29	+ 1.42	— 0.50	— 4.06	— 3.48	— 0.08	— 0.03	— 0.16	+ 0.16	— 0.203	+ 0.008	— 0.063	177
).04	+ 2.43	— 0.31	+ 3.84	— 9.99	+ 0.07	+ 0.07	— 0.28	0.00	— 0.038	— 0.004	+ 0.159	192
7.05	— 6.48	+ 0.37	— 3.25	+ 8.90	+ 0.06	— 0.01	— 0.07	— 0.14	+ 0.055	+ 0.004	— 0.018	212
;.38	— 5.39	+ 0.66	— 1.88	+ 5.86	0.00	+ 0.02	0.00	— 0.07	+ 0.109	+ 0.003	+ 0.104	223
;.34	— 5.33	+ 0.67	— 1.66	+ 5.73	0.00	+ 0.02	0.00	— 0.07	+ 0.117	+ 0.003	+ 0.108	230
).51	— 3.78	+ 0.12	+ 3.52	— 11.09	+ 0.09	+ 0.13	— 0.42	— 0.09	+ 0.573	+ 0.009	— 0.061	261
).57	— 5.63	+ 0.15	— 2.48	— 11.08	— 0.69	+ 0.32	— 1.82	— 1.20	+ 0.442	+ 0.013	— 0.002	277
7.88	+ 6.58	— 0.31	+ 2.52	+ 9.89	— 0.03	— 0.14	— 0.23	— 0.32	— 0.313	— 0.010	+ 0.031	288
;.81	+ 5.92	— 0.46	— 3.68	+ 7.01	+ 0.02	0.00	— 0.01	— 0.06	— 0.222	+ 0.002	— 0.010	312
4.82	+ 4.10	— 0.47	— 1.17	+ 3.32	0.00	— 0.01	0.00	+ 0.02	— 0.124	— 0.001	+ 0.018	323
3.43	— 0.75	— 0.44	— 1.22	— 7.37	— 0.04	+ 0.05	— 0.17	+ 0.08	+ 0.688	+ 0.006	— 0.031	353
;.13	+ 4.79	+ 0.66	— 0.10	+ 5.08	+ 0.01	— 0.03	— 0.01	— 0.08	— 0.249	— 0.004	— 0.110	375
).54	+ 1.56	+ 0.27	— 2.83	— 9.28	— 0.12	+ 0.04	— 0.35	+ 0.04	— 0.173	— 0.008	+ 0.154	396
;.52	— 0.86	+ 0.75	— 1.95	— 2.71	— 0.02	+ 0.02	— 0.05	+ 0.07	+ 0.754	— 0.002	+ 0.272	472
).49	— 3.01	+ 0.13	+ 1.66	— 9.91	+ 0.01	+ 0.10	— 0.29	— 0.06	+ 0.593	+ 0.013	— 0.051	492
;.60	— 0.65	— 0.40	+ 2.54	— 3.42	0.00	+ 0.01	— 0.02	+ 0.02	+ 0.211	— 0.003	— 0.208	511
;.30	+ 5.08	— 0.06	+ 0.89	+ 6.73	— 0.08	+ 0.09	— 0.13	— 0.30	— 0.432	— 0.007	— 0.128	544

FINLAY

h_4	Δk_4	y_4	$\Delta\varepsilon_2$	$\Delta\varepsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
7.61	— 6.81	— 0.32	— 3.01	+ 9.82	+ 0.06	+ 0.17	— 0.26	— 0.43	— 0.625	+ 0.008	— 0.441	88
0.22	+ 3.99	+ 0.14	— 2.48	+ 10.95	+ 0.42	— 0.21	— 1.25	— 0.40	— 1.115	— 0.013	— 0.031	359
9.87	— 3.24	— 0.14	+ 2.22	+ 10.35	— 0.18	— 0.97	— 2.20	— 0.57	— 0.084	+ 0.012	— 0.133	402
9.20	— 5.07	— 0.18	— 3.48	+ 10.44	+ 0.15	0.00	— 0.34	— 0.25	+ 0.019	+ 0.008	— 0.048	416
7.37	— 5.73	— 0.34	— 0.81	+ 8.83	— 0.04	+ 0.21	— 0.31	— 0.50	— 0.271	+ 0.011	— 0.117	420
5.41	— 1.29	— 0.33	— 3.37	— 1.83	— 0.04	— 0.05	— 0.10	+ 0.17	+ 0.387	— 0.007	+ 0.127	454
9.55	+ 4.54	+ 0.02	+ 1.61	+ 10.57	— 0.01	— 0.26	— 0.66	— 0.33	— 0.720	— 0.015	+ 0.062	456
4.31	— 1.58	— 0.30	— 2.41	— 0.68	— 0.02	— 0.05	— 0.07	+ 0.13	+ 0.456	— 0.005	+ 0.321	536

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TABLE XIII. SATELLITES II and IV

OBSERV

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coeffi... Δp_2	Δq_2	Δp_4	Δq_4	x_2
	1891			h m	° ′ ″	° ′ ″						
5	Aug. 17	2	2	9 45.11	238 0 41	238 11 37	1.0	— 3.17	+ 2.45	+ 10.38	+ 4.34	+ 0.05
23	19	2-3	3	10 14.70	241 7 40	241 20 3	1.0	+ 3.86	— 0.98	+ 4.55	+ 10.31	+ 0.04
29	20	2-3	3	8 18.54	244 2 43	244 3 49	1.0	+ 0.67	+ 3.95	+ 0.77	+ 11.25	+ 0.10
57	24	2-3	2-3	8 41.04	246 24 58	246 22 24	1.0	— 2.47	+ 3.22	— 11.18	+ 1.21	+ 0.10
77	25	2-3	2-3	8 38.96	261 29 12	260 52 33	1.0	— 2.66	— 2.87	— 10.41	— 2.91	— 0.37
94	29	2-3	3	7 59.84	64 39 3	64 39 1	1.0	+ 0.04	+ 4.07	— 2.22	+ 11.21	— 0.10
114	31	3-4	3-4	8 7.37	66 58 22	66 47 56	0.25	+ 1.71	— 3.69	— 9.28	+ 6.76	— 0.08
137	Sept. 1	—		8 34.13	66 5 16	66 2 0	1.0	+ 3.30	+ 2.44	— 11.14	+ 2.86	— 0.02
157	5	2	2-3	8 45.90	244 4 7	244 0 34	1.0	— 0.54	— 4.04	+ 3.60	+ 10.82	+ 0.12
167	7	1-2	2-3	7 36.83	245 8 54	245 9 36	1.0	— 0.86	+ 3.97	— 4.61	+ 10.35	+ 0.09
176	8	—		6 51.97	246 24 45	246 19 28	1.0	— 3.88	— 1.35	— 8.02	+ 7.97	+ 0.01
191	10	2	2-3	7 13.87	246 43 46	246 43 15	1.0	+ 2.88	+ 2.82	— 11.29	+ 0.14	— 0.01
211	13	2	3	7 55.61	64 33 59	64 33 39	1.0	— 3.96	+ 0.66	+ 4.80	+ 10.29	— 0.06
222	14	3-4 & 4	3-4 & 4	6 51.78	65 1 52	64 56 4	0.25	— 0.09	— 4.05	+ 0.89	+ 11.36	— 0.09
260	19	2-3	3	7 17.86	67 36 5	67 33 39	1.0	+ 2.29	+ 3.35	— 11.13	— 2.45	+ 0.05
276	20	1-2	2	6 47.80	76 28 30	76 5 5	1.0	— 3.56	+ 1.64	— 9.32	— 6.14	+ 0.05
287	21	1	1-2	6 28.15	243 28 14	243 33 1	1.0	+ 0.92	+ 3.91	+ 6.56	+ 9.22	+ 0.14
311	22	2	2-3	7 20.08	245 9 41	245 8 3	1.0	— 4.06	— 0.04	+ 2.54	+ 10.98	+ 0.04
322	23	2	2-3	6 42.31	245 37 20	245 33 22	1.0	+ 0.58	— 3.96	— 1.60	+ 11.11	+ 0.07
352	Oct. 4	3	3	6 19.94	66 35 2	66 31 7	1.0	— 2.63	+ 2.89	— 10.16	+ 4.61	— 0.04
374	9	2-3	3	9 47.84	245 15 31	245 15 38	1.0	— 1.68	+ 3.55	+ 0.76	+ 10.88	+ 0.06
395	13	2-3	2-3	9 46.72	247 37 20	247 31 38	1.0	— 3.66	+ 1.33	— 10.65	+ 1.25	— 0.02
471	Nov. 5	2-3	3-4	8 24.86	66 19 21	66 16 21	1.0	— 2.96	+ 2.01	— 6.30	+ 8.04	+ 0.01
491	8	2	2-3	7 30.18	67 29 27	67 28 10	1.0	+ 0.30	+ 3.55	— 9.92	— 1.96	+ 0.04
510	22	3	3-4	7 43.16	65 55 36	65 53 4	0.5	+ 1.43	+ 3.09	— 6.73	+ 6.94	0.00
543	Dec. 14	2	3	6 32.08	243 22 19	243 25 42	1.0	+ 2.10	— 2.36	+ 3.55	+ 8.20	+ 0.04

OBSERV

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coeffi... Δp_2	Δq_2	Δp_4	Δq_4	x_2
	1891			h m	° ′ ″	° ′ ″						
89	Aug. 27	3-4	3	8 24.10	62 34 34	62 29 51	0.25	— 1.36	— 3.79	+ 5.89	+ 9.68	— 0.14
358	Oct. 6	2-3	3	5 37.44	238 53 10	239 12 35	1.0	— 3.50	+ 1.79	+ 10.50	+ 3.33	+ 0.15
401	14	2-3	3	9 0.73	53 31 42	54 14 7	1.0	+ 0.85	+ 3.66	+ 10.15	+ 2.61	— 0.57
415	15	3	3	8 56.21	63 30 0	63 32 58	0.5	— 3.80	+ 0.03	+ 8.65	+ 6.32	— 0.09
419	16	2-3	3	8 48.23	62 38 34	62 34 21	1.0	+ 0.73	— 3.78	+ 5.73	+ 9.05	— 0.14
453	19	2	3	10 1.47	66 50 9	66 41 45	1.0	— 2.43	— 2.90	— 5.83	+ 9.01	+ 0.03
455	23	2	3	8 6.28	241 51 51	242 4 33	1.0	+ 0.16	+ 3.76	+ 9.48	+ 4.71	+ 0.19
535	Dec. 8	3	3-4	7 19.35	66 29 16	66 22 37	0.5	— 1.26	— 3.01	— 4.46	+ 8.00	+ 0.04

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)SITION-ANGLES

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	y_2	Δh_4	Δk_4	y_4	$\Delta\varepsilon_2$	$\Delta\varepsilon_4$	O—C	(O—C) × s sin 1″	Differential aberration	Residuals	rot. Nr.
							′ ″	″	″	″	
14	+ 0.01	— 1.17	+ 0.76	+ 0.03	— 0.08	+ 1.37	+ 10 56	+ 0.420	— 0.002	— 0.092	5
10	0.00	— 0.43	+ 0.71	+ 0.03	+ 0.10	+ 0.82	+ 12 23	+ 0.810	— 0.001	+ 0.476	23
11	+ 0.01	— 0.04	+ 0.37	+ 0.01	+ 0.11	+ 0.38	+ 1 6	+ 0.229	0.000	— 0.057	29
07	— 0.01	+ 0.34	+ 0.09	— 0.01	+ 0.12	+ 0.35	— 2 34	— 0.451	0.000	+ 0.168	57
05	— 0.11	+ 2.96	— 1.04	— 0.03	+ 1.09	+ 3.13	— 36 39	— 0.866	+ 0.001	+ 0.133	77
10	+ 0.01	— 0.04	— 0.32	0.00	+ 0.10	+ 0.32	— 0 2	— 0.008	0.000	— 0.042	94
06	+ 0.01	— 0.37	— 0.17	+ 0.01	— 0.10	+ 0.40	— 10 26	— 1.096	0.000	— 0.189	114
00	+ 0.01	— 0.28	— 0.13	+ 0.01	— 0.08	+ 0.30	— 3 16	— 0.457	0.000	— 0.125	137
12	— 0.01	— 0.11	+ 0.38	— 0.01	— 0.12	+ 0.40	— 3 33	— 0.271	0.000	+ 0.020	157
09	— 0.01	+ 0.10	+ 0.27	0.00	+ 0.10	+ 0.29	+ 0 42	+ 0.178	0.000	+ 0.324	167
00	— 0.01	+ 0.26	+ 0.20	0.00	+ 0.12	+ 0.33	— 5 18	— 1.002	0.000	— 0.003	176
01	0.00	+ 0.34	+ 0.08	— 0.01	— 0.09	+ 0.34	— 0 31	— 0.055	0.000	+ 0.202	191
06	0.00	+ 0.10	— 0.32	+ 0.01	+ 0.04	+ 0.33	— 0 20	— 0.052	0.000	+ 0.023	211
09	+ 0.01	0.00	— 0.30	+ 0.01	— 0.09	+ 0.29	— 5 48	— 0.599	0.000	— 0.135	222
04	— 0.01	— 0.45	+ 0.01	— 0.01	— 0.11	+ 0.44	— 2 26	— 0.188	— 0.001	+ 0.070	260
02	0.00	— 1.88	+ 0.94	— 0.04	+ 0.58	+ 2.08	— 23 25	— 0.621	— 0.002	+ 0.144	276
14	— 0.01	— 0.28	+ 0.43	0.00	+ 0.15	+ 0.51	+ 4 47	+ 0.689	— 0.001	+ 0.003	287
04	0.00	— 0.02	+ 0.28	+ 0.01	+ 0.05	+ 0.27	— 1 38	— 0.287	0.000	— 0.010	311
08	0.00	+ 0.07	+ 0.25	0.00	— 0.08	+ 0.26	— 3 58	— 0.455	0.000	+ 0.074	322
06	0.00	— 0.25	— 0.14	0.00	+ 0.11	+ 0.29	— 3 55	— 0.790	0.000	— 0.105	352
08	+ 0.01	— 0.01	+ 0.27	+ 0.01	+ 0.09	+ 0.27	+ 0 7	+ 0.026	0.000	— 0.059	374
04	— 0.02	+ 0.39	+ 0.07	— 0.01	+ 0.17	+ 0.39	— 5 42	— 0.766	+ 0.001	+ 0.063	395
05	0.00	— 0.13	— 0.17	0.00	+ 0.09	+ 0.21	— 3 0	— 0.643	0.000	— 0.070	471
01	0.00	— 0.30	— 0.01	— 0.01	0.00	+ 0.29	— 1 17	— 0.111	0.000	+ 0.199	491
03	0.00	— 0.10	— 0.15	0.00	+ 0.02	+ 0.17	— 2 32	— 0.477	0.000	— 0.214	510
06	0.00	— 0.20	+ 0.34	— 0.01	— 0.03	+ 0.39	+ 3 23	+ 0.161	0.000	+ 0.096	543

LAY

	y_2	Δh_4	Δk_4	y_4	$\Delta\varepsilon_2$	$\Delta\varepsilon_4$	O—C	(O—C) × s sin 1″	Differential aberration	Residuals	rot. Nr.
							′ ″	″	″	″	
17	— 0.01	+ 0.34	— 0.52	0.00	— 0.18	+ 0.62	— 4 43	— 0.235	+ 0.001	— 0.051	89
08	+ 0.02	— 1.19	+ 0.60	+ 0.03	— 0.21	+ 1.31	+ 19 25	+ 0.600	— 0.002	+ 0.158	358
46	— 0.05	+ 2.11	— 0.82	— 0.02	+ 0.53	+ 2.24	+ 42 25	+ 1.033	+ 0.002	+ 0.005	401
06	0.00	+ 0.37	— 0.34	0.00	— 0.08	+ 0.50	+ 2 58	+ 0.194	+ 0.001	+ 0.001	415
13	— 0.01	+ 0.42	— 0.49	— 0.01	— 0.12	+ 0.64	— 4 13	— 0.161	+ 0.001	— 0.136	419
00	0.00	— 0.18	— 0.18	+ 0.01	+ 0.05	+ 0.25	— 8 24	— 1.240	0.000	— 0.304	453
17	+ 0.01	— 0.66	+ 0.44	— 0.02	+ 0.17	+ 0.78	+ 12 42	+ 0.861	— 0.001	+ 0.028	455
01	0.00	— 0.13	— 0.14	+ 0.01	+ 0.01	+ 0.19	— 6 39	— 0.770	0.000	— 0.015	535

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TABLE XIV. SATELLITES III and IV.

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficient μ	Coefficient z_G	Coefficient x_3	Coefficient Δh_3	Coefficient Δk_3	Coefficient y_3
	1891			h m	"	"								
8	Aug. 17	2	2	10 2.29	211.687	211.302	c	1.0	— 2.12	— 0.47	— 2.09	+ 3.02	— 3.49	+ 0.71
16	19	2-3	3	8 53.68	552.851	553.184	c	1.0	— 5.53	— 0.18	+ 0.46	+ 5.60	— 1.63	+ 0.54
40	20	2	3	9 40.66	403.145	403.585	c	1.0	— 4.03	— 0.25	— 2.15	+ 4.60	— 3.96	+ 0.72
46	21	3-4	3-4	8 20.08	284.483	284.673	c	0.25	— 2.84	— 0.35	— 1.66	+ 2.61	— 2.67	+ 0.50
54	24	2-3	2-3	8 12.10	622.682	622.575	c	1.0	— 6.23	— 0.16	— 2.61	+ 3.61	— 3.84	+ 0.35
70	25	3	3	7 54.05	528.527	528.232	c	0.5	— 5.29	— 0.19	— 0.64	+ 2.86	— 1.50	+ 0.37
97	29	2-3	3	8 15.85	964.973	965.126	c	1.0	— 9.65	— 0.10	— 0.16	— 3.87	+ 0.76	— 0.53
113	31	3-4	3-4	7 52.96	387.558	387.902	c	0.25	— 3.88	— 0.26	+ 2.97	— 4.09	+ 3.90	— 0.32
127	Sept. 1	—		7 14.32	88.372	88.819	ψ	1.0	— 0.88	0.00	+ 1.33	— 2.66	+ 1.89	— 0.15
147	5	3	3-4	7 12.24	83.684	83.290	ψ	0.5	— 0.84	0.00	— 0.37	+ 3.40	— 0.87	— 0.21
170	7	1-2	2-3	7 47.62	817.021	817.234	c	1.0	— 8.17	— 0.12	— 3.26	+ 4.56	— 3.82	— 0.80
178	8	—		7 6.66	976.301	976.357	c	1.0	— 9.76	— 0.10	+ 0.14	+ 2.71	— 2.24	— 0.48
181		—		7 31.56	976.259	*976.221	c	2.0	— 9.76	— 0.10	+ 0.18	+ 2.70	— 2.19	— 0.48
189	10	2	2-3	6 51.26	329.446	329.856	c	1.0	— 3.29	— 0.30	— 2.28	+ 5.85	— 2.75	— 0.77
210	13	2	3	7 45.36	590.328	590.203	c	1.0	— 5.90	— 0.17	+ 1.03	— 5.66	+ 1.26	+ 0.51
225	14	3-4 &4	3-4 &4	7 4.56	442.659	442.684	c	0.25	— 4.43	— 0.23	+ 3.46	— 5.04	+ 3.60	+ 0.72
229		1-2	1-2	7 33.91	438.953	438.834	c	1.0	— 4.39	— 0.23	+ 3.48	— 4.99	+ 3.62	+ 0.72
257	19	1-2	2	6 50.68	525.181	525.691	c	1.0	— 5.25	— 0.19	+ 1.51	— 2.90	+ 1.40	+ 0.39
275	20	1-2	2	6 39.14	151.083	151.492	c	1.0	— 1.51	— 0.66	+ 1.07	— 5.21	+ 0.81	+ 0.77
290	21	1	1-3	6 39.47	386.025	385.918	c	1.0	— 3.86	— 0.26	— 3.57	+ 5.42	— 3.24	— 0.66
306	22	2	2-3	6 48.43	825.644	825.653	c	1.0	— 8.26	— 0.12	— 3.10	+ 2.98	— 2.88	— 0.29
319	23	2	2-3	6 11.65	956.366	956.620	c	1.0	— 9.56	— 0.10	— 0.90	+ 3.72	— 0.58	— 0.52
324				6 53.09	954.993	955.278	c	1.0	— 9.55	— 0.10	— 0.88	+ 3.80	— 0.54	— 0.53
337	29	2	3	5 35.44	34.986	35.066	ψ	1.0	— 0.35	0.00	— 3.68	+ 3.28	— 3.18	+ 0.29
351	Oct. 4	3	3	6 9.69	775.040	775.634	c	0.5	— 7.75	— 0.13	+ 1.38	— 4.42	+ 0.38	— 0.30
371	9	2-3	3	9 21.85	406.371	406.788	c	1.0	— 4.06	— 0.25	— 4.62	+ 4.77	— 3.37	+ 0.68
383	10	2-3	3	9 12.40	276.298	276.433	c	1.0	— 2.76	— 0.36	— 2.83	+ 2.73	— 2.11	+ 0.47
394	13	2-3	2-3	9 32.32	564.654	564.694	c	1.0	— 5.65	— 0.18	— 4.24	+ 3.57	— 3.22	+ 0.38
476	Nov. 5	2-3	3-4	8 55.59	285.229	285.200	c	1.0	— 2.85	— 0.35	+ 2.55	— 3.95	+ 0.22	+ 0.50
488	6	3	3& 3-4	8 26.12	422.846	422.903	c	0.5	— 4.23	— 0.24	+ 4.76	— 5.41	+ 1.83	+ 0.65
496	8	2-3	2-3 &3	8 0.20	439.522	439.852	c	1.0	— 4.40	— 0.23	+ 2.49	— 2.66	+ 1.05	+ 0.36
499	9	—		7 20.64	111.913	112.554	c	0.5	— 1.12	— 0.89	+ 3.35	— 4.65	+ 0.49	+ 0.71
507	22	2	2-3	7 12.79	819.647	820.040	c	1.0	— 8.20	— 0.12	+ 3.03	— 2.41	+ 1.85	— 0.40
518	27	2	2-3	6 53.34	500.915	500.821	c	1.0	— 5.01	— 0.20	— 4.06	+ 4.61	— 0.63	+ 0.35
540	Dec. 14	2-3	3	5 57.22	44.414	44.239	ψ	1.0	— 0.44	0.00	— 2.55	+ 2.41	— 0.85	— 0.15

OBSERVE

rot. Nr.	Date	Definition I.	Definition S.	Greenwich time	Computed distance	Observed distance	method	weight	Coefficient μ	Coefficient z_F	Coefficient x_3	Coefficient Δh_3	Coefficient Δk_3	Coefficient y_3
	1891			h m	"	"								
86	Aug. 27	3-4	3	8 2.20	401.281	400.218	c	0.25	— 4.01	— 0.25	+ 2.18	— 5.21	+ 3.64	— 0.59
245	Sept. 18	2	3	7 25.88	616.237	616.435	c	1.0	— 6.16	— 0.16	+ 3.76	— 3.85	+ 3.67	+ 0.40
357	Oct. 6	2-3	3	5 22.27	178.743	178.342	c	1.0	— 1.79	— 0.56	— 4.11	+ 3.65	— 3.30	+ 0.73
408	14	3	3-4	9 43.39	438.645	438.584	c	0.5	— 4.39	— 0.23	— 1.90	+ 3.00	— 0.90	+ 0.43
414	15	3	3	8 39.76	71.602	71.759	ψ	0.5	— 0.72	0.00	— 2.49	+ 5.06	— 0.69	+ 0.75
432	16	3	3-4	10 3.10	466.273	465.637	c	0.5	— 4.66	— 0.21	+ 4.79	— 5.00	+ 3.13	— 0.61
438	17	3	3	8 58.89	830.371	830.261	c	0.5	— 8.30	— 0.12	+ 3.34	— 2.81	+ 2.43	— 0.30
452	19	2	3	9 49.00	674.142	674.211	c	1.0	— 6.74	— 0.15	+ 3.99	— 5.68	+ 1.80	— 0.73
460	23	2	3	8 38.14	5.954	4.478	ψ	0.0	— 0.06	0.00	+ 3.46	— 3.76	+ 1.96	+ 0.03
534	Dec. 8	3	3-4	7 7.04	638.128	638.356	c	0.5	— 6.38	— 0.16	+ 4.70	— 4.72	+ 1.17	— 0.63

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DISTANCES.

GILL

Δh_4	Δk_4	y_4	$\Delta\varepsilon_3$	$\Delta\varepsilon_4$	Δp_3	Δq_3	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
− 9.97	+ 5.10	+ 0.16	+ 3.85	+ 11.18	− 0.10	− 0.72	− 1.19	− 0.50	− 0.385	− 0.012	+ 0.026	8
− 6.77	+ 7.08	+ 0.40	− 5.62	+ 9.24	+ 0.04	0.00	− 0.03	− 0.06	+ 0.333	− 0.002	+ 0.096	16
− 4.99	+ 5.87	+ 0.70	− 5.94	+ 6.04	− 0.03	− 0.04	0.00	+ 0.09	+ 0.440	+ 0.006	− 0.010	40
− 4.54	+ 3.92	+ 0.86	− 2.17	+ 2.28	+ 0.01	− 0.07	− 0.04	+ 0.11	+ 0.190	0.000	+ 0.016	46
− 9.45	+ 2.18	+ 0.26	+ 4.82	− 9.11	+ 0.02	+ 0.06	− 0.11	+ 0.01	− 0.107	− 0.005	− 0.158	54
− 10.23	+ 4.04	+ 0.10	− 0.16	− 10.92	− 0.10	+ 0.17	− 0.33	− 0.09	− 0.295	− 0.016	− 0.331	70
+ 4.70	− 4.33	− 0.46	− 2.61	+ 3.36	0.00	0.00	0.00	+ 0.01	+ 0.153	0.000	+ 0.124	97
+ 7.01	− 1.24	− 0.15	− 5.35	− 4.92	− 0.14	− 0.27	− 0.44	+ 0.32	+ 0.344	− 0.006	+ 0.126	113
+ 8.89	− 1.62	− 0.03	− 0.79	− 8.15	+ 0.42	− 1.06	− 1.95	+ 0.54	+ 0.447	+ 0.009	+ 0.069	127
− 6.33	+ 6.57	− 0.11	+ 1.66	+ 8.28	− 0.66	+ 0.72	− 0.59	− 1.62	− 0.394	− 0.009	− 0.175	147
− 4.92	+ 2.94	− 0.39	+ 5.76	+ 0.63	− 0.04	− 0.06	+ 0.05	− 0.11	+ 0.213	+ 0.010	+ 0.020	170
− 6.31	+ 1.40	− 0.50	+ 1.56	− 3.53	− 0.01	+ 0.02	− 0.03	+ 0.03	+ 0.056	− 0.002	− 0.031	178
− 6.34	+ 1.38	− 0.50	+ 1.47	− 3.60	− 0.01	+ 0.03	− 0.03	+ 0.03	− 0.038	− 0.002	− 0.121	181
− 9.99	+ 2.41	− 0.31	− 6.46	− 9.93	0.46	− 0.20	− 0.88	+ 0.02	+ 0.410	− 0.001	+ 0.124	189
+ 7.07	− 6.48	+ 0.37	− 5.57	+ 8.92	+ 0.03	0.00	− 0.02	− 0.04	− 0.125	+ 0.001	− 0.039	210
+ 5.36	− 5.39	+ 0.66	− 6.11	+ 5.83	− 0.04	− 0.04	+ 0.01	+ 0.10	+ 0.025	− 0.005	+ 0.215	225
+ 5.34	− 5.34	+ 0.67	− 6.07	+ 5.76	− 0.04	− 0.04	+ 0.01	+ 0.10	− 0.119	− 0.007	+ 0.068	229
+ 10.51	− 3.74	+ 0.13	− 0.02	− 11.08	− 0.08	+ 0.17	− 0.32	− 0.07	+ 0.510	+ 0.017	− 0.061	257
+ 9.65	− 5.65	+ 0.15	− 4.86	− 11.16	− 0.93	+ 0.23	− 1.42	− 0.93	+ 0.409	+ 0.011	+ 0.055	275
− 7.85	+ 6.58	− 0.31	+ 6.28	+ 9.86	− 0.30	− 0.22	− 0.38	− 0.54	− 0.107	− 0.001	+ 0.032	290
− 5.84	+ 5.94	− 0.46	+ 3.04	+ 7.08	0.00	− 0.10	− 0.04	− 0.18	+ 0.009	− 0.004	− 0.036	306
− 4.83	+ 4.15	− 0.47	− 2.28	+ 3.41	0.00	0.00	0.00	0.00	+ 0.254	− 0.001	+ 0.063	319
− 4.82	+ 4.08	− 0.47	− 2.43	+ 3.29	0.00	0.00	0.00	+ 0.01	+ 0.285	0.000	+ 0.090	324
− 8.43	+ 5.94	0.00	+ 3.85	− 10.08	+ 0.25	+ 1.08	− 1.31	− 1.45	+ 0.080	− 0.009	+ 0.062	337
+ 8.42	− 0.76	− 0.44	− 3.61	− 7.35	− 0.11	+ 0.07	− 0.21	+ 0.10	+ 0.594	+ 0.003	+ 0.039	351
− 5.15	+ 4.83	+ 0.65	− 5.70	+ 5.15	− 0.04	− 0.05	+ 0.01	+ 0.11	+ 0.417	+ 0.007	+ 0.015	371
− 4.80	+ 2.82	+ 0.83	− 1.72	+ 1.22	+ 0.01	− 0.09	− 0.04	+ 0.15	+ 0.135	+ 0.001	+ 0.017	383
− 9.53	+ 1.53	+ 0.27	+ 4.27	− 9.27	+ 0.02	+ 0.06	− 0.12	+ 0.01	+ 0.040	− 0.007	+ 0.025	394
+ 5.56	− 0.84	+ 0.75	+ 3.13	− 2.79	+ 0.03	− 0.02	− 0.04	+ 0.05	− 0.029	− 0.002	− 0.109	476
+ 7.30	− 0.40	+ 0.54	+ 5.70	− 6.08	+ 0.02	0.00	− 0.02	+ 0.01	+ 0.057	− 0.007	− 0.011	488
+ 9.49	− 3.05	+ 0.13	− 0.15	− 9.92	− 0.06	+ 0.14	− 0.26	− 0.05	+ 0.330	+ 0.015	− 0.190	496
+ 8.66	− 4.69	+ 0.14	− 4.32	− 9.82	− 0.80	+ 0.24	− 1.25	− 0.79	+ 0.641	+ 0.008	+ 0.324	499
+ 5.57	− 0.67	− 0.41	+ 1.55	− 3.35	0.00	+ 0.02	− 0.02	+ 0.03	+ 0.393	+ 0.001	+ 0.175	507
− 6.14	+ 5.29	+ 0.26	− 4.38	+ 7.59	+ 0.02	0.00	− 0.02	− 0.03	− 0.094	− 0.003	− 0.232	518
− 5.29	+ 5.03	− 0.06	+ 0.54	+ 6.70	− 0.39	+ 0.64	− 0.53	− 1.20	− 0.175	− 0.008	+ 0.011	540

FINLAY

Δh_4	Δk_4	y_4	$\Delta\varepsilon_3$	$\Delta\varepsilon_4$	Δp_3	Δq_3	Δp_4	Δq_4	O—C	Differential aberration	Residuals	rot. Nr.
									"	"	"	
+ 7.64	− 6.79	− 0.32	+ 6.31	+ 9.84	− 0.30	− 0.26	− 0.37	− 0.60	− 1.063	+ 0.001	− 0.441	86
+ 9.77	− 1.82	+ 0.30	+ 4.88	− 9.39	+ 0.02	+ 0.06	− 0.11	+ 0.02	+ 0.198	+ 0.004	− 0.038	245
− 10.21	+ 3.97	+ 0.14	+ 4.41	+ 10.94	− 0.28	− 0.70	− 1.28	− 0.40	− 0.401	− 0.010	+ 0.192	357
− 10.08	+ 3.41	+ 0.13	− 0.73	− 10.61	− 0.10	+ 0.17	− 0.33	− 0.09	− 0.061	− 0.016	− 0.002	408
− 8.88	+ 4.91	+ 0.17	− 4.81	− 10.10	− 1.63	+ 0.34	− 2.36	− 1.70	+ 0.157	− 0.007	− 0.013	414
+ 7.27	− 5.73	− 0.31	+ 5.82	+ 8.72	− 0.18	− 0.16	− 0.22	− 0.36	− 0.636	− 0.001	− 0.100	432
+ 5.55	− 4.98	− 0.47	+ 2.46	+ 5.99	0.00	− 0.07	− 0.02	− 0.12	− 0.110	+ 0.003	+ 0.086	438
+ 5.40	− 1.30	− 0.33	− 5.93	+ 1.79	− 0.11	− 0.02	− 0.11	+ 0.17	+ 0.069	− 0.011	− 0.009	452
+ 6.77	− 3.33	0.00	+ 4.23	− 7.54	+ 3.65	+ 2.16	− 6.68	− 3.39	− 1.476	+ 0.003	[−0.966]	460
+ 4.31	− 1.59	− 0.30	− 4.76	− 0.66	− 0.07	0.00	− 0.06	+ 0.11	+ 0.228	− 0.009	+ 0.194	534

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TABLE XV. SATELLITES III and IV.

OBSERVE

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficient Δp_3	Coefficient Δq_3	Coefficient Δp_4	Coefficient Δq_4	Coefficient x_3
	1891			h m	° ′ ″	° ′ ″						
7	Aug. 17	2	2	10 2.08	238 29 28	238 33 8	1.0	+ 0.83	+ 6.30	+ 10.37	+ 4.40	− 0.4
15	19	2-3	3	8 53.12	244 42 56	244 44 28	1.0	− 6.44	− 0.07	+ 4.79	+ 10.24	− 0.1
39	20	2	3	9 40.72	245 31 16	245 34 33	1.0	− 3.92	− 5.11	+ 0.53	+ 11.26	− 0.0
45	21	3-4	3-4	8 19.55	245 40 54	245 41 42	0.25	+ 1.12	− 6.35	− 3.46	+ 10.71	− 0.1
53	24	2-3	2-3	8 12.37	245 39 0	245 31 35	1.0	+ 2.11	+ 6.08	− 11.17	+ 1.30	− 0.1
69	25	3	3	7 53.58	246 51 14	246 45 10	0.5	− 3.28	+ 5.56	− 10.88	− 2.90	− 0.1
96	29	2-3	3	8 15.98	65 12 45	65 10 54	1.0	− 5.12	+ 3.94	− 2.28	+ 11.20	+ 0.1
112	31	3-4	3-4	7 52.36	67 55 16	67 52 3	0.25	− 2.99	− 5.72	− 9.25	+ 6.79	− 0.0
126	Sept. 1	—		7 15.47	75 21 39	75 2 29	1.0	+ 2.33	− 5.94	− 10.91	+ 3.04	− 0.0
146	5	3	3-4	7 13.28	236 35 48	236 48 3	0.5	+ 4.32	− 4.73	+ 3.84	+ 10.61	− 0.2
169	7	1-2	2-3	7 47.62	244 40 39	244 38 54	1.0	+ 3.76	+ 5.26	− 4.64	+ 10.34	− 0.1
182	8	—		7 40.10	245 32 5	245 29 23	1.0	− 1.62	+ 6.27	− 8.12	+ 7.86	− 0.1
209	13	2	3	7 45.39	65 6 47	65 7 26	1.0	− 6.44	+ 0.44	+ 4.84	+ 10.28	+ 0.0
224	14	3-4 &4	3-4 &4	7 4.37	65 52 13	65 53 55	0.25	− 4.55	− 4.57	+ 0.85	+ 11.36	+ 0.0
256	19	1-2	2	6 49.84	67 5 28	66 58 4	1.0	− 2.82	+ 5.78	− 11.15	− 2.37	+ 0.1
274	20	1-2	2	6 38.53	74 1 9	73 47 23	1.0	− 6.15	+ 1.53	− 9.40	− 6.17	− 0.0
289	21	1	1-2	6 38.83	242 7 52	242 7 57	1.0	+ 5.14	+ 3.80	+ 6.52	+ 9.24	− 0.2
305	22	2	2-3	6 47.93	244 33 38	244 33 0	1.0	+ 0.32	+ 6.39	+ 2.63	+ 10.96	− 0.2
318	23	2	2-3	6 11.34	245 31 25	245 29 14	1.0	− 4.63	+ 4.44	− 1.50	+ 11.12	− 0.1
336	29	2	3	5 36.28	255 42 19	254 25 13	1.0	+ 1.42	+ 6.07	− 7.32	− 8.14	+ 0.5
350	Oct. 4	3	3	6 9.20	66 50 10	66 45 55	1.0	− 5.40	+ 3.20	− 10.15	+ 4.64	+ 0.0
370	9	2-3	3	9 21.68	246 16 37	246 19 17	1.0	− 4.20	− 4.61	+ 0.84	+ 10.88	− 0.0
382	10	2-3	3	9 12.83	246 30 46	246 33 1	1.0	+ 0.83	− 6.15	− 3.21	+ 10.37	− 0.0
393	13	2-3	2-3	9 32.90	246 21 14	246 15 12	1.0	+ 2.15	+ 5.76	− 10.66	+ 1.29	− 0.0
475	Nov. 5	2-3	3-4	8 55.70	66 9 40	66 4 11	1.0	+ 4.76	− 3.26	− 6.37	+ 7.98	+ 0.0
487	6	3	3 & 3-4	8 26.91	65 59 28	65 53 51	0.5	+ 5.57	+ 1.47	− 8.77	+ 5.19	0.0
495	8	2-3	2-3 & 3	7 58.94	67 18 51	67 12 13	1.0	− 2.35	+ 5.20	− 9.90	− 2.03	− 0.0
506	22	2	2-3	7 11.62	65 58 12	65 53 34	1.0	− 0.67	+ 5.41	− 6.67	+ 6.99	+ 0.0
517	27	2	2-3	6 52.92	245 29 44	245 30 27	1.0	− 5.26	+ 1.09	+ 4.69	+ 8.20	0.0
539	Dec. 14	2-3	3	5 55.58	237 0 20	237 25 7	1.0	+ 2.62	− 4.29	+ 3.58	+ 8.09	− 0.3

OBSERVE

rot. Nr	Date	Definition I.	Definition S.	Greenwich time	Computed position-angle	Observed position-angle	weight	Coefficient Δp_3	Coefficient Δq_3	Coefficient Δp_4	Coefficient Δq_4	Coefficient x_3
	1891			h m	° ′ ″	° ′ ″						
87	Aug. 27	3-4	3	8 2.10	61 35 55	61 43 49	0.0	+ 4.86	+ 4.27	+ 5.93	+ 9.64	+ 0.2
244	Sept. 18	2	3	7 25.81	65 59 26	65 50 46	1.0	+ 2.53	+ 5.95	− 11.30	+ 1.75	+ 0.0
356	Oct. 6	2-3	3	5 23.20	238 43 14	238 43 20	1.0	+ 2.27	+ 5.77	+ 10.51	+ 3.29	− 0.0
407	14	3	3-4	9 43.85	247 34 24	247 26 49	0.5	− 3.10	+ 5.31	− 10.34	− 2.79	− 0.0
413	15	3	3	8 39.26	261 27 8	261 18 3	0.5	− 5.79	+ 1.20	− 8.36	− 6.05	+ 0.0
431	16	3	3-4	10 3.72	63 32 9	63 32 31	0.5	+ 4.59	+ 4.05	+ 5.56	+ 9.16	+ 0.2
437	17	3	3	8 59.53	65 6 8	65 4 46	0.5	+ 0.06	+ 6.10	+ 1.99	+ 10.55	+ 0.3
451	19	2	3	9 48.70	66 52 2	66 50 2	1.0	− 5.92	− 1.25	− 5.80	+ 9.03	− 0.3
459	23	2	3	8 37.92	110 44 40	103 8 46	0.0	+ 3.66	+ 2.18	− 6.71	− 3.40	− 3.3
533	Dec. 8	3	3-4	7 7.02	66 21 23	66 18 20	0.5	− 5.16	− 0.19	− 4.43	+ 8.02	− 0.3

87 Rejected; included in first approximation (with weight 0.25) $n = (O - C)s \sin 1''$ from Cape solution = − 0″.050 (see page 58).

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POSITION-ANGLES.

GILL

of Δh_3	Δk_3	y_3	Δh_4	Δk_4	y_4	$\Delta \varepsilon_3$	$\Delta \varepsilon_4$	O—C	(O—C) × $s \sin 1''$	Differential aberration	Residuals	rot. Nr
								′ ″	″	″	″	
+ 0.30	— 0.55	+ 0.10	— 1.08	+ 0.72	+ 0.03	+ 0.59	+ 1.28	+ 3 40	+ 0.227	— 0.001	+ 0.100	7
— 0.07	— 0.09	+ 0.01	— 0.02	+ 0.27	+ 0.01	+ 0.07	+ 0.25	+ 1 32	+ 0.247	0.000	+ 0.053	15
— 0.07	— 0.07	0.00	+ 0.09	+ 0.22	0.00	— 0.02	+ 0.21	+ 3 17	+ 0.384	0.000	+ 0.125	39
— 0.10	— 0.14	0.00	+ 0.15	+ 0.23	— 0.01	— 0.15	+ 0.27	+ 0 48	+ 0.066	0.000	— 0.027	45
— 0.07	— 0.10	— 0.01	+ 0.21	+ 0.12	— 0.01	+ 0.07	+ 0.23	— 7 25	— 1.344	0.000	— 0.200	53
— 0.19	— 0.12	— 0.03	+ 0.38	+ 0.01	— 0.01	+ 0.20	+ 0.37	— 6 4	— 0.932	+ 0.001	+ 0.095	69
+ 0.13	+ 0.13	+ 0.03	— 0.08	— 0.28	0.00	+ 0.18	+ 0.29	— 1 51	— 0.519	0.000	+ 0.054	96
+ 0.22	— 0.06	+ 0.03	— 0.49	— 0.15	+ 0.01	+ 0.16	+ 0.48	— 3 13	— 0.363	0.000	— 0.055	112
+ 0.56	— 0.17	+ 0.05	— 1.73	+ 0.13	+ 0.02	— 0.05	+ 1.64	— 19 10	— 0.493	— 0.002	— 0.071	126
+ 0.41	— 0.28	— 0.05	— 0.95	+ 1.24	— 0.03	+ 0.07	+ 1.49	+ 12 15	+ 0.298	— 0.002	— 0.107	146
+ 0.02	— 0.15	— 0.02	+ 0.06	+ 0.30	— 0.01	+ 0.13	+ 0.29	— 1 45	— 0.416	0.000	+ 0.378	169
— 0.08	— 0.16	— 0.02	+ 0.17	+ 0.22	— 0.01	+ 0.17	+ 0.27	— 2 42	— 0.766	0.000	+ 0.268	182
+ 0.07	+ 0.09	+ 0.01	+ 0.03	— 0.25	+ 0.01	+ 0.08	+ 0.24	+ 0 39	+ 0.112	0.000	— 0.034	209
+ 0.07	+ 0.06	0.00	— 0.08	— 0.22	0.00	+ 0.01	+ 0.21	+ 1 42	+ 0.219	0.000	+ 0.002	224
+ 0.17	+ 0.12	— 0.03	— 0.36	— 0.02	— 0.01	+ 0.18	+ 0.35	— 7 24	— 1.132	— 0.001	— 0.027	256
+ 0.89	— 0.03	— 0.14	— 1.47	+ 0.70	+ 0.01	+ 0.86	+ 1.61	— 13 46	— 0.605	— 0.002	+ 0.082	274
+ 0.28	— 0.27	— 0.03	— 0.46	+ 0.58	— 0.01	+ 0.39	+ 0.74	+ 0 5	+ 0.009	0.000	— 0.002	289
+ 0.01	— 0.20	+ 0.01	— 0.08	+ 0.34	0.00	+ 0.20	+ 0.35	— 0 38	— 0.152	0.000	+ 0.265	305
— 0.10	— 0.14	+ 0.02	+ 0.06	+ 0.26	0.00	+ 0.17	+ 0.26	— 2 11	— 0.608	0.000	— 0.073	318
— 0.61	+ 0.43	— 0.04	+ 1.49	— 0.88	+ 0.02	— 0.56	+ 1.66	— 77 6	— 0.786	+ 0.001	+ 0.016	336
— 0.19	+ 0.10	— 0.01	— 0.29	— 0.14	0.00	+ 0.22	+ 0.32	— 4 15	— 0.959	0.000	+ 0.010	350
— 0.07	— 0.06	0.00	+ 0.08	+ 0.19	0.00	0.00	+ 0.18	+ 2 40	+ 0.315	0.000	+ 0.068	370
— 0.09	+ 0.12	0.00	+ 0.14	+ 0.20	— 0.01	— 0.13	+ 0.23	+ 2 15	+ 0.181	0.000	+ 0.092	382
— 0.06	— 0.08	— 0.01	+ 0.17	+ 0.11	0.00	+ 0.06	+ 0.18	— 6 2	— 0.992	0.000	+ 0.101	393
+ 0.10	+ 0.09	— 0.01	— 0.11	— 0.17	0.00	— 0.14	+ 0.20	— 5 29	— 0.454	0.000	— 0.145	475
+ 0.06	+ 0.05	— 0.01	— 0.10	— 0.14	0.00	— 0.05	+ 0.16	— 5 37	— 0.691	0.000	+ 0.065	487
+ 0.14	+ 0.09	— 0.02	— 0.27	— 0.03	— 0.01	+ 0.14	+ 0.26	— 6 38	— 0.848	0.000	+ 0.134	495
+ 0.05	+ 0.11	— 0.01	— 0.10	— 0.15	0.00	+ 0.11	+ 0.18	— 4 38	— 1.106	0.000	— 0.195	506
— 0.05	— 0.06	+ 0.01	— 0.03	+ 0.17	0.00	+ 0.06	+ 0.17	+ 0 43	+ 0.104	0.000	— 0.025	517
+ 0.30	— 0.23	— 0.03	— 0.79	+ 0.90	— 0.02	— 0.04	+ 1.14	+ 24 47	+ 0.320	— 0.001	— 0.077	539

FINLAY

of Δh_3	Δk_3	y_3	Δh_4	Δk_4	y_4	$\Delta \varepsilon_3$	$\Delta \varepsilon_4$	O—C	(O—C) × $s \sin 1''$	Differential aberration	Residuals	rot. Nr
								′ ″	″	″	″	
— 0.28	+ 0.32	— 0.02	+ 0.47	— 0.63	— 0.01	+ 0.42	+ 0.78	+ 7 54	+ 0.924	0.000	[+ 1.033]	87
+ 0.06	+ 0.09	— 0.01	— 0.20	— 0.12	— 0.01	+ 0.06	+ 0.22	— 8 40	— 1.555	0.000	— 0.368	244
+ 0.43	— 0.52	+ 0.11	— 1.22	+ 0.61	+ 0.03	+ 0.64	+ 1.34	+ 0 6	+ 0.005	— 0.001	— 0.175	356
— 0.17	— 0.11	— 0.03	+ 0.35	+ 0.01	— 0.01	+ 0.18	+ 0.33	— 7 35	— 0.969	0.000	+ 0.009	407
— 1.52	+ 0.11	— 0.23	+ 2.49	— 1.24	— 0.06	+ 1.46	+ 2.75	— 9 5	— 0.189	+ 0.002	+ 0.418	413
— 0.17	+ 0.20	— 0.02	+ 0.30	— 0.41	0.00	+ 0.26	+ 0.51	+ 0 22	+ 0.050	0.000	+ 0.144	431
0.00	+ 0.16	+ 0.01	+ 0.06	— 0.28	0.00	+ 0.16	+ 0.29	— 1 22	— 0.330	0.000	+ 0.129	437
+ 0.16	+ 0.03	+ 0.03	— 0.19	— 0.18	+ 0.01	+ 0.15	+ 0.25	— 2 0	— 0.392	0.000	+ 0.011	451
+ 3.78	— 1.85	— 0.01	— 6.74	+ 3.14	+ 0.02	— 4.20	+ 7.43	— 455 54	— 0.787	— 0.003	[— 0.204]	459
+ 0.12	+ 0.03	+ 0.02	— 0.12	— 0.14	0.00	+ 0.12	+ 0.18	— 3 3	— 0.565	0.000	— 0.190	533

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CHAPTER III.

From the equations of condition normal equations were formed for each pair of satellites, and these partial normals were added to form the complete normal equations. For the distances the observations by the two observers were treated separately. Thus, if it appears by the residuals that the observations of one

NORMA

A. DISTANC

μ	z_0	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	$\Delta\varepsilon_4$	x_1	Δh_1	Δk_1	x_2	Δh_2	Δk_2	x_3	Δh_3
+ 2534.8	+ 125.2	+ 66.7	— 32.2	+ 78.0	— 118.1	— 10.7	+ 33.6	— 17.1	— 47.1	— 39.8	+ 58.4	+ 128.8	— 251.8 +
	+ 35.6	+ 6.2	— 12.4	+ 3.7	— 0.7	— 5.6	+ 8.6	— 8.4	— 14.3	— 16.1	+ 15.7	+ 12.5	+ 1.1 +
		+ 179.3	— 25.7	— 41.6	+ 48.7	— 11.4	+ 38.9	— 23.6	— 12.9	+ 4.3	+ 13.4	+ 15.2	— 23.4 +
			+ 485.9	+ 43.9	— 37.9	— 6.2	+ 9.5	— 9.4	— 48.2	— 85.0	+ 56.2	— 5.9	— 19.8 —
				+ 1290.7	— 132.9	+ 1.8	— 55.2	+ 9.7	— 11.9	— 53.2	+ 9.3	— 147.5	+ 222.1 —
					+ 4502.7	— 29.4	+ 53.9	— 9.5	+ 75.4	+ 114.1	— 65.0	— 137.0	+ 226.2 —
						+ 53.8	— 77.1	+ 42.5	+ 26.0	+ 42.9	— 22.0	— 46.5	+ 64.8 —
							+ 189.0	— 73.4	— 50.6	— 100.8	+ 51.1	+ 85.7	— 150.4 +
								+ 48.9	+ 18.0	+ 42.0	— 22.8	— 40.5	+ 67.0 —
									+ 171.6	+ 219.5	— 152.5	+ 66.2	— 111.6 +
										+ 470.1	— 231.1	+ 96.2	— 210.1 +
											+ 166 5	— 56.8	+ 118.4 —
												+ 483.9	— 636.8 +
													+ 1265.3 —
													+

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of the observers are affected by some personality, it will be easier to form a judgment about the influence of that error on the final result, than it would be if the observations of both observers had been treated collectively. For the position-angles we must suppose, as has been remarked before, that there is no personality, so there was no reason to separate the observations by the two observers.

The products were multiplied out to their full extent, so as to make the check by sums absolutely rigorous. The computations were done by Messrs. S. L. Goodman and V. A. Löwinger of the Cape Observatory. The normals thus derived are given below. The absolute terms n include the corrections for differential aberration. The equations C are the sums of the corresponding equations A and B; similarly set E is derived by adding the corresponding equations of C and D. (N.B. These additions were done before the last decimals were dropped.)

QUATIONS.

LL.

Δh_4	Δk_4	y_2	y_3	y_4	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	n	
587.4 —	475.5 —	2.6 +	14.1 —	1.2 +	3.0 +	0.4 +	6.7 +	3.2 +	17.0 +	9.3 +	56.1 +	23.5	— 28".034	(μ
22.6 —	21.9 +	1.0 —	3.5 —	2.5 +	1.0 —	0.6 +	2.2 +	1.8 +	6.1 +	2.7 +	8.4 +	4.6	— 0.047	(ε_0
26.0 +	16.4 +	2.5 +	0.6 +	3.7 —	0.9 —	0.7 —	5.8 —	3.7 +	1.3 +	1.4 +	1.6 —	3.7	+ 13.562	($\Delta\varepsilon_1$
83.8 +	6.2 —	10.2 +	6.8 —	1.6 —	5.9 —	3.9 +	1.1 —	1.2 —	16.5 —	13.4 +	15.0 +	3.3	+ 3.570	($\Delta\varepsilon_2$
210.2 +	108.0	0.0 —	43.6 —	16.6 +	3.4 —	2.2 —	18.5 —	9.6 +	24.3 —	7.1 +	4.2 —	10.1	— 25.668	($\Delta\varepsilon_3$
977.4 +	943.8 —	9.4 —	0.3 +	15.6 —	2.0 +	2.4 +	17.6 —	2.6 +	5.8 —	14.2 +	114.4 —	33.8	— 89.853	($\Delta\varepsilon_4$
09.9 —	48.9 +	1.1 —	1.1 —	0.6 —	0.5 +	2.1 —	2.5 —	0.5 —	0.9 —	1.0 +	10.2 +	4.2	+ 12.015	(x_1
29.3 +	134.2 —	1.3 +	4.0 —	0.2 —	2.2 —	2.4 +	5.4 —	0.2 +	7.3 +	6.4 —	14.9 —	7.2	— 19.993	(Δh_1
98.2 —	47.2 +	1.2 —	0.3 —	2.0 +	0.4 +	3.6 —	3.2 —	0.8 —	2.9 —	2.3 +	15.0 +	5.7	+ 8.676	(Δk_1
28.9 +	121.1 +	0.6 +	7.1 —	4.0 +	0.1 —	0.7 —	7.3 —	6.1 —	10.7 —	1.7 —	2.7 —	1.4	— 15.488	(x_2
452.4 +	241.5 +	11.5 +	6.8 —	5.0 +	2.5 —	2.0 —	10.8 —	14.2 —	15.8 +	3.8 —	8.8 —	3.5	— 30.940	(Δh_2
238.1 —	132.9 —	2.0 —	4.7 +	2.2 —	1.6 —	0.1 +	8.3 +	7.8 +	10.7 +	1.3 +	7.3 +	3.2	+ 16.928	(Δk_2
461.6 —	251.3 +	2.2 +	18.4 +	0.8 +	2.3 —	3.2 +	0.4 —	5.7 —	4.3 +	8.2 +	5.2 +	10.6	— 24.414	(x_3
342.7 +	483.5 —	2.8 —	42.0 —	8.0 +	0.1 +	1.6 —	0.5 +	13.5 +	4.5 —	16.7 —	0.8 —	10.3	+ 49.613	(Δh_3
458.6 —	249.0 +	3.4 +	9.0 +	1.2 +	1.9 —	3.3 +	1.5 —	5.8 +	5.6 +	15.3 +	7.6 +	10.2	— 26.339	(Δk_3
364.1 —	2098.2 —	9.3 +	23.8 +	16.5 +	3.8 +	26.2 +	2.6 +	15.8 —	0.3 —	7.9 +	146.3 +	83.9	+ 140.135	(Δh_4
+	1494.9 +	1.2 —	9.8 —	6.1 —	2.0 —	11.4 —	0.6 —	7.2 +	0.6 +	4.6 —	73.4 —	55.0	— 57.841	(Δk_4
	+	3.1 —	0.1 —	0.3 —	0.3 +	0.2 +	0.2 —	0.8 —	0.1 +	0.4 —	1.7 —	0.4	— 0.778	(y_2
		+	21.0 +	6.3 —	0.4 +	0.3 —	1.3 —	0.1 —	6.5 —	3.2 —	1.7 —	0.9	— 0.222	(y_3
			+	13.3 —	0.1 —	0.3 —	0.2	0.0 +	0.1 —	0.1 —	2.6 —	1.1	+ 0.574	(y_4
				+	5.1 +	1.8 +	3.6 +	1.3 +	0.7 +	2.5 +	6.5 +	2.6	+ 0.501	(Δp_1
					+	10.7 +	4.7 +	0.7 —	0.5 —	2.4 +	29.3 +	10.6	+ 3.752	(Δq_1
						+	17.4 +	3.6 +	5.1 +	4.8 +	3.2 +	1.5	+ 1.191	(Δp_2
							+	6.8 +	3.1 —	2.4 +	2.3 +	0.5	+ 2.523	(Δq_2
								+	25.7 +	12.3 +	1.8 +	2.2	— 0.922	(Δp_3
									+	20.2 +	0.6 —	3.4	— 2.462	(Δq_3
										+	154.3 +	56.9	+ 12.229	(Δp_4
											+	28.7	+ 6.052	(Δq_4

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B. DISTANC

μ	z_F	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	$\Delta\varepsilon_4$	x_1	Δh_1	Δk_1	x_2	h_2	Δk_2	x_3	Δh_3
+ 459.7	+ 30.0	+ 16.5	— 4.6	— 69.8	— 96.8	— 34.1	+ 40.8	— 14.9	— 5.1	— 12.9	+ 3.0	— 147.2	+ 153.9 —
	+ 10.5	— 0.5	— 5.0	— 10.9	— 34.2	— 1.1	+ 1.0	— 0.9	— 4.8	— 6.2	+ 2.6	— 10.6	+ 9.0 —
		+ 48.5	— 1.9	+ 6.2	— 40.8	— 22.9	+ 27.1	— 9.6	— 14.4	— 12.3	+ 10.1	+ 10.6	— 15.1 +
			+ 89.2	— 21.2	— 12.5	+ 6.6	— 5.0	+ 2.9	+ 47.4	+ 41.9	— 30.6	+ 31.4	— 39.8 +
				+ 410.9	+ 89.8	— 0.3	— 6.0	— 7.4	— 1.9	+ 10.1	+ 1.0	+ 8.0	— 2.7 +
					+ 1253.6	+ 31.7	— 29.8	+ 22.6	— 14.7	— 2.3	+ 16.1	— 31.1	+ 4.7 —
						+ 34.4	— 38.4	+ 17.9	+ 13.6	+ 12.0	— 9.2	— 31.7	+ 33.8 —
							+ 48.0	— 19.0	— 16.5	— 14.4	+ 11.4	+ 37.7	— 40.7 +
								+ 10.9	+ 5.7	+ 5.3	— 3.9	— 17.5	+ 18.4 —
									+ 80.9	+ 77.4	— 54.8	+ 47.0	— 56.8 +
										+ 84.8	— 51.0	+ 50.2	— 60.8 +
											+ 38.1	— 31.9	+ 38.1 —
												+ 274.5	— 305.7 +
													+ 357.5 —
													+

C. DISTANC

μ	z_G	z_F	$\Delta\varepsilon_1$	$\Delta\varepsilon_2$	$\Delta\varepsilon_3$	$\Delta\varepsilon_4$	x_1	Δh_1	Δk_1	x_2	Δh_2	Δk_2	x_3
+ 2994.5	+ 125.2	+ 30.0	+ 83.2	— 36.8	+ 8.2	— 214.9	— 44.8	+ 74.4	— 32.0	— 52.2	— 52.7	+ 61.4	— 18.4 —
	+ 35.6	—	+ 6.2	— 12.4	+ 3.7	— 0.7	— 5.6	+ 8.6	— 8.4	— 14.3	— 16.1	+ 15.7	+ 12.5 +
		+ 10.5	— 0.5	— 5.0	— 10.9	— 34.2	— 1.1	+ 1.0	— 0.9	— 4.8	— 6.2	+ 2.6	— 10.6 +
			+ 227.8	— 27.6	— 35.4	+ 7.9	— 34.4	+ 66.0	33.2	— 27.3	— 8.0	+ 23.5	+ 25.8 —
				+ 575.1	+ 22.7	— 50.5	+ 0.4	+ 4.5	— 6.5	— 0.8	— 43.1	+ 25.6	+ 25.4 —
					+ 1701.6	— 43.1	+ 1.5	— 61.3	+ 2.3	— 13.8	— 43.0	+ 10.3	— 139.4 +
						+ 5756.3	+ 2.3	+ 24.1	+ 13.1	+ 60.7	+ 111.9	— 48.9	— 168.2 +
							+ 88.2	— 115.4	+ 60.4	+ 39.6	+ 54.9	— 31.3	— 78.2 +
								+ 237.0	— 92.4	— 67.1	— 115.2	+ 62.5	+ 123.2 —
									+ 59.8	+ 23.7	+ 47.2	— 26.7	— 58.0 +
										+ 252.6	+ 296.8	— 207.4	+ 113.2 —
											+ 554.8	— 282.1	+ 146.4 —
												+ 204.6	— 88.6 +
													+ 758.4 —
													+

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NLAY.

h_4	Δk_4	y_2	y_3	y_4	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	n	
67.2	+ 133.0	+ 1.4	+ 3.2	+ 14.9	— 0.1	+ 0.8	— 0.3	+ 2.9	+ 6.2	+ 4.2	+ 16.2	+ 5.0	+ 8".615	(μ
0.7	+ 4.2	+ 0.3	0.0	+ 0.6	— 0.2	+ 0.6	— 0.4	+ 0.5	+ 2.5	+ 0.9	+ 2.8	+ 1.3	+ 4.685	(z_F
24.9	+ 9.6	— 0.1	— 0.7	+ 0.7	+ 0.4	+ 0.1	— 0.1	— 0.5	— 1.0	— 1.1	— 3.2	— 0.8	+ 2.349	($\Delta\varepsilon_1$
18.8	+ 14.8	— 3.9	— 5.3	+ 1.7	— 0.3	— 0.1	— 1.6	— 2.6	— 0.8	— 0.4	— 1.4	— 0.4	— 5.022	($\Delta\varepsilon_2$
24.6	— 25.8	+ 1.9	+ 20.0	+ 2.3	— 0.5	— 2.3	+ 0.2	— 2.6	— 5.0	— 10.9	— 0.9	— 1.0	— 15.694	($\Delta\varepsilon_3$
52.0	— 151.5	+ 0.6	— 5.1	— 15.2	— 2.0	— 3.4	+ 3.3	— 10.9	+ 4.0	— 12.3	— 46.5	— 23.8	— 46.298	($\Delta\varepsilon_4$
53.4	— 30.3	— 0.2	+ 3.4	— 2.6	— 0.8	+ 0.1	0.0	+ 0.2	— 0.1	+ 0.3	+ 4.3	+ 0.8	+ 3.448	(x_1
74.4	+ 37.6	+ 0.1	— 4.7	+ 2.8	+ 1.9	— 0.4	0.0	— 0.4	+ 0.5	+ 0.2	— 9.4	— 2.4	— 5.488	(Δh_1
27.8	— 18.2	— 0.1	+ 1.3	— 1.4	— 0.2	0.0	0.0	+ 0.1	— 0.1	+ 0.3	+ 1.4	0.0	+ 1.884	(Δk_1
00.1	+ 44.6	— 0.7	— 5.5	+ 2.8	+ 0.5	— 0.2	+ 1.1	+ 1.0	— 1.3	— 0.7	+ 4.1	+ 0.9	— 10.897	(x_2
18.3	+ 53.6	— 0.3	— 4.6	+ 3.0	+ 0.4	— 0.2	+ 1.9	+ 0.1	— 1.1	— 0.8	+ 1.1	+ 0.3	— 13.288	(Δh_2
65.3	— 29.9	+ 0.4	+ 3.6	— 1.9	— 0.4	+ 0.2	— 0.6	— 0.9	+ 0.9	+ 0.4	— 3.7	— 0.9	+ 6.647	(Δk_2
60.7	— 66.5	— 1.4	— 27.8	— 3.5	+ 0.2	+ 0.4	+ 0.7	— 1.3	+ 1.2	+ 0.5	+ 6.7	+ 3.5	— 21.759	(x_3
87.8	+ 79.5	+ 1.4	+ 33.8	+ 4.4	+ 0.1	— 1.0	— 1.0	+ 1.4	— 4.0	+ 0.8	— 8.8	— 5.3	+ 24.372	(Δh_3
13.9	— 47.1	— 0.7	— 12.4	— 1.6	+ 0.2	— 0.1	+ 0.4	— 1.0	+ 0.1	+ 0.7	+ 3.8	+ 1.1	— 13.846	(Δk_3
64.6	— 526.5	— 4.7	— 18.3	— 26.0	— 10.8	+ 3.7	— 5.7	— 1.7	+ 8.7	+ 4.0	+ 64.1	+ 20.6	+ 35.758	(Δh_4
	+ 286.3	+ 2.0	+ 9.6	+ 15.3	+ 3.8	— 1.4	+ 2.0	— 0.8	— 4.0	— 0.7	— 24.8	— 4.6	— 10.238	(Δk_4
		+ 0.4	+ 0.1	0.0	0.0	0.0	+ 0.2	+ 0.1	0.0	0.0	+ 0.1	0.0	— 0.008	(y_2
			+ 5.9	+ 0.9	0.0	— 0.2	— 0.1	0.0	— 1.1	— 0.5	— 1.7	— 0.9	+ 1.777	(y_3
				+ 1.4	+ 0.1	0.0	+ 0.1	0.0	— 0.1	0.0	— 0.6	— 0.1	— 0.495	(y_4
					+ 0.7	— 0.2	0.0	0.0	— 0.1	0.0	— 3.0	— 0.9	— 0.689	(Δp_1
						+ 0.2	0.0	0.0	+ 0.3	0.0	+ 0.8	+ 0.3	+ 0.453	(Δq_1
							+ 0.2	+ 0.1	0.0	0.0	— 0.1	— 0.1	— 0.554	(Δp_2
								+ 0.9	+ 0.1	0.0	+ 2.1	+ 0.5	+ 0.675	(Δq_2
									+ 2.5	+ 0.3	+ 2.3	+ 1.5	+ 0.707	(Δp_3
										+ 0.9	+ 0.5	+ 0.1	+ 0.704	(Δq_3
											+ 25.7	+ 8.8	+ 5.650	(Δp_4
												+ 4.0	+ 2.321	(Δq_4

LL AND FINLAY.

3	Δh_4	Δk_4	y_2	y_3	y_4	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	n	
2.7	+ 320.2	— 342.5	— 1.2	+ 17.3	+ 13.7	+ 2.9	+ 1.2	+ 6.4	+ 6.1	+ 23.2	+ 13.5	+ 72.3	+ 28.5	— 19".419	(μ
9.8	+ 22.6	— 21.9	+ 1.0	— 3.5	— 2.5	+ 1.0	— 0.6	+ 2.2	+ 1.8	+ 6.1	+ 2.7	+ 8.4	+ 4.6	— 0.047	(z_G
5.2	+ 0.7	+ 4.2	+ 0.3	0.0	+ 0.6	— 0.2	+ 0.6	— 0.4	+ 0.5	+ 2.5	+ 0.9	+ 2.8	+ 1.3	+ 4.685	(z_F
5.8	— 50.9	+ 26.0	+ 2.4	— 0.1	+ 4.4	— 0.5	— 0.6	— 5.9	— 4.2	+ 0.3	+ 0.4	— 1.6	— 4.5	+ 15.911	($\Delta\varepsilon_1$
1.9	+ 65.0	+ 21.0	— 14.1	+ 1.6	+ 0.1	— 6.2	— 4.0	— 0.5	— 3.8	— 17.3	— 13.7	+ 13.6	+ 2.9	— 1.452	($\Delta\varepsilon_2$
2.5	— 185.6	+ 82.2	+ 1.9	— 23.5	— 14.3	+ 3.0	— 4.5	— 18.3	— 12.3	+ 19.2	— 18.0	+ 3.2	— 11.1	— 41.362	($\Delta\varepsilon_3$
4.1	— 1025.4	+ 792.3	— 8.8	— 5.4	+ 0.4	— 4.0	— 1.1	+ 20.9	— 13.5	+ 9.8	— 26.5	+ 67.9	— 57.6	— 136.151	($\Delta\varepsilon_4$
5.1	+ 163.3	— 79.2	+ 0.9	+ 2.3	— 3.2	— 1.3	+ 2.2	— 2.5	— 0.3	— 1.0	— 0.8	+ 14.5	+ 5.1	+ 15.463	(x_1
5.0	— 303.7	+ 171.8	— 1.2	— 0.6	+ 2.7	— 0.2	— 2.9	+ 5.4	— 0.7	+ 7.8	+ 6.5	— 24.4	— 9.6	— 25.481	(Δh_1
3.8	+ 125.9	— 65.4	+ 1.1	+ 1.0	— 3.4	+ 0.2	+ 3.6	— 3.2	— 0.7	— 2.9	— 2.0	+ 16.4	+ 5.7	+ 10.560	(Δk_1
8.6	— 328.9	+ 165.6	— 0.1	+ 1.6	— 1.2	+ 0.6	— 0.9	— 6.2	— 5.1	— 12.0	— 2.3	+ 1.4	— 0.5	— 26.385	(x_2
6.8	— 570.7	+ 295.1	+ 11.2	+ 2.2	— 2.0	+ 2.9	— 2.2	— 8.9	— 14.1	— 16.9	+ 3.0	— 7.7	— 3.2	— 44.228	(Δh_2
7.0	+ 303.4	— 162.8	— 1.6	— 1.2	+ 0.3	— 2.0	0.0	+ 7.7	+ 6.8	+ 11.6	+ 1.7	+ 3.6	+ 2.2	+ 23.575	(Δk_2
0.3	+ 622.3	— 317.7	+ 0.9	— 9.4	— 2.7	+ 2.5	— 2.8	+ 1.1	— 7.0	— 3.1	+ 8.7	+ 11.9	+ 14.0	— 46.173	(x_3
6.5	— 1030.4	+ 563.0	— 1.3	— 8.2	— 3.6	+ 0.2	+ 0.6	— 1.4	+ 14.9	+ 0.4	— 15.9	— 9.6	— 15.6	+ 73.985	(Δh_3
7.1	+ 572.5	— 296.1	+ 2.6	— 3.4	— 0.4	+ 2.1	— 3.4	+ 1.9	— 6.8	+ 5.7	+ 16.1	+ 11.5	+ 11.4	— 40.185	(Δk_3
	+ 5328.7	— 2624.6	— 14.0	+ 5.4	— 9.5	— 7.0	+ 29.9	— 3.1	+ 14.1	+ 8.4	— 3.9	+ 210.4	+ 104.5	+ 175.893	(Δh_4
		+ 1781.2	+ 3.2	— 0.2	+ 9.2	+ 1.8	— 12.8	+ 1.4	— 8.0	— 3.4	+ 3.9	— 98.3	— 59.7	— 68.079	(Δk_4
			+ 3.5	0.0	— 0.3	— 0.3	+ 0.2	+ 0.4	— 0.7	— 0.1	+ 0.4	— 1.6	— 0.3	— 0.786	(y_2
				+ 26.8	+ 7.2	— 0.4	+ 0.1	— 1.4	— 0.1	— 7.6	— 3.7	— 3.4	— 1.8	+ 1.555	(y_3
					+ 14.7	0.0	— 0.3	— 0.1	0.0	0.0	0.0	— 3.2	— 1.1	+ 0.079	(y_4
						+ 5.8	+ 1.6	+ 3.6	+ 1.4	+ 0.6	+ 2.6	+ 3.4	+ 1.8	— 0.188	(Δp_1
							+ 10.9	+ 4.7	+ 0.7	— 0.2	— 2.4	+ 30.1	+ 10.8	+ 4.205	(Δq_1
								+ 17.7	+ 3.6	+ 5.1	+ 4.8	+ 3.1	+ 1.4	+ 0.637	(Δp_2
									+ 7.7	+ 3.1	— 2.3	+ 4.4	+ 1.0	+ 3.198	(Δq_2
										+ 28.2	+ 12.5	+ 4.1	+ 3.7	— 0.215	(Δp_3
											+ 21.1	+ 1.2	— 3.3	— 1.758	(Δq_3
												+ 180.0	+ 65.7	+ 17.879	(Δp_4
													+ 32.7	+ 8.373	(Δq_4

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D. POSITION ANGLE

Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	$\Delta \varepsilon_1$	$\Delta \varepsilon_2$	$\Delta \varepsilon_3$	$\Delta \varepsilon_4$	x_1
+ 281.6	— 38.9	— 21.2	— 52.8	— 20.6	+ 19.8	— 30.2	— 36.0	— 6.5	+ 12.2	— 1.2	+ 4.4	+ 1.5
	+ 245.2	+ 10.2	— 27.0	— 44.1	— 62.0	+ 36.7	+ 38.9	+ 8.6	+ 2.0	+ 1.7	+ 3.0	— 2.3
		+ 591.8	— 40.4	— 42.7	+ 88.5	+ 42.9	— 129.8	+ 12.0	— 18.8	+ 22.6	— 24.1	+ 1.7
			+ 783.8	+ 42.6	— 41.0	— 101.4	+ 65.2	+ 4.6	+ 18.2	+ 4.1	+ 16.3	+ 0.9
				+ 1484.0	+ 21.2	+ 137.5	— 186.2	— 1.1	+ 16.3	— 41.5	— 6.9	+ 1.3
					+ 1927.4	— 301.0	+ 75.6	— 2.2	+ 11.0	+ 71.0	+ 30.9	+ 1.9
						+ 5106.5	+ 511.6	+ 5.1	— 16.1	— 4.0	— 74.7	— 16.5
							+ 5604.4	+ 6.4	+ 0.7	+ 16.6	+ 187.6	— 6.2
								+ 11.6	+ 7.8	+ 0.9	+ 16.5	+ 3.2
									+ 29.1	+ 6.3	+ 5.8	+ 7.6
										+ 42.5	+ 5.0	+ 0.6
											+ 192.7	— 2.9
												+ 4.6

E. DISTANCES AND POSITI

Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	μ	z_0	z_F	$\Delta \varepsilon_1$	$\Delta \varepsilon_2$	$\Delta \varepsilon_3$
+ 287.4	— 37.3	— 17.6	— 51.5	— 20.0	+ 22.4	— 26.8	— 34.3	+ 2.9	+ 1.0	— 0.2	— 6.9	+ 6.0	+ 1.8
	+ 256.2	+ 14.8	— 26.3	— 44.3	— 64.4	+ 66.9	+ 49.8	+ 1.2	— 0.6	+ 0.6	+ 8.0	— 2.0	— 2.8
		+ 609.4	— 36.7	— 37.5	+ 93.2	+ 46.0	— 128.4	+ 6.4	+ 2.2	— 0.4	+ 6.2	— 19.3	+ 4.3
			+ 791.5	+ 45.7	— 43.2	— 97.0	+ 66.1	+ 6.1	+ 1.8	+ 0.5	+ 0.4	+ 14.4	— 8.2
				+ 1512.2	+ 33.7	+ 141.6	— 182.5	+ 23.2	+ 6.1	+ 2.5	— 0.8	— 1.0	— 22.3
					+ 1948.5	— 299.8	+ 72.3	+ 13.5	+ 2.7	+ 0.9	— 1.8	— 2.7	+ 53.0
						+ 5286.4	+ 577.3	+ 72.3	+ 8.4	+ 2.8	+ 3.5	— 2.5	— 0.8
							+ 5637.1	+ 28.5	+ 4.6	+ 1.3	+ 1.9	+ 3.5	+ 5.4
								+ 2994.5	+ 125.2	+ 30.0	+ 83.2	— 36.8	+ 8.2
									+ 35.6	—	+ 6.2	— 12.4	+ 3.7
										+ 10.5	— 0.5	— 5.0	— 10.9
											+ 239.4	— 19.8	— 34.5
												+ 604.2	+ 29.0
													+ 1744.1

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GILL AND FINLAY.

Δk_1	x_2	Δh_2	Δk_2	x_3	Δh_3	Δk_3	Δh_4	Δk_4	y_2	y_3	y_4	n	
1.4 —	0.6 —	8.5 +	4.3 —	2.6 +	0.8 —	1.8 +	7.9 +	1.0 +	0.9 +	0.8 —	0.1	+ 3".785	(Δp_1
3.8 +	0.3 +	5.8	0.0 +	3.1 —	2.4 +	4.2 —	31.0 +	13.9 —	0.3 —	0.6 +	0.2	— 4.328	(Δq_1
4.2 +	2.8 +	20.2 —	8.0 —	6.4 +	5.4 —	7.1 +	2.2 —	2.0 —	1.3 +	1.3	0.0	+ 39.245	(Δp_2
1.4 +	6.8 +	13.9 —	9.2 +	6.9 —	18.1 —	8.6 —	14.3 +	8.3 +	0.6 —	0.5	0.0	+ 56.314	(Δq_2
2.4 +	8.9 +	14.8 —	9.1 —	4.9 —	2.9 —	11.0 —	5.6 +	6.9 —	0.3 +	6.2 —	0.1	+ 8.366	(Δp_3
1.9 +	1.0 -	4.8 +	0.2 —	22.8 +	18.7 —	29.4 +	8.1 —	2.6 —	0.1 +	3.6 +	0.1	— 148.091	(Δq_3
19.4 —	0.1 +	5.3 —	6.0 —	16.1 +	5.5 —	16.6 —	196.2 +	125.8 +	1.3 +	5.2 +	4.6	+ 327.871	(Δp_4
6.7 +	4.4 +	0.5 —	6.1 —	21.6 +	13.7 —	15.5 —	112.6 +	86.3 +	0.2 +	2.9 +	2.3	-- 58.496	(Δq_4
2.4 +	5.1 +	12.4 —	4.0 +	0.6 +	0.6 +	0.8 —	8.1 +	4.5 —	0.2 —	0.1 +	0.2	+ 2.026	($\Delta \varepsilon_1$
5.2 +	10.3 +	12.4 —	5.1 +	2.0 —	5.9 +	2.4 +	3.5 —	0.9 +	0.6 +	0.8 —	0.1	+ 1.178	($\Delta \varepsilon_2$
0.7 +	2.4 +	1.1 —	1.9 +	0.4 —	14.6 +	4.0 —	1.9 +	1.2	0.0 —	0.2	0.0	— 6.349	($\Delta \varepsilon_3$
6.3 —	1.8 —	1.6 +	2.4 —	0.7 +	1.0 —	1.5 —	28.5 +	24.9 —	0.4 —	0.4 +	0.7	— 12.789	($\Delta \varepsilon_4$
4.0 +	4.3 +	6.3 —	2.4 —	0.2 —	0.4	0.0 +	10.7 —	5.1 +	0.1 —	0.1 --	0.2	+ 0.758	(x_1
5.3 —	4.5 -	11.4 +	3.8 +	0.9 —	3.6 +	2.0 --	15.9 +	7.0 +	0.2 —	0.4 +	0.2	— 1.968	(Δh_1
4.4 +	3.2 +	5.8 —	2.2 —	0.4 +	0.7 —	0.7 +	15.2 —	7.5	0.0 +	0.1 —	0.2	+ 1.036	(Δk_1
+	7.5 +	10.7 +	5.2 +	1.8 —	3.4 +	2.2 —	2.9 +	1.5 +	0.2 +	0.3	0.0	+ 1.640	(x_2
	+	26.5 —	10.0 +	3.8 —	7.8 +	4.9 —	5.8 +	2.2 —	0.2 +	0.4	0.0	+ 5.190	(Δh_2
		+	5.1 —	2.4 +	4.4 —	3.1 +	3.4 —	1.8 —	0.1 —	0.3	0.0	— 2.320	(Δk_2
			+	8.8 —	11.3 +	10.4 +	3.4 —	2.7 +	0.2 —	0.4	0.0	+ 0.050	(x_3
				+	31.0 —	17.3 —	6.8 +	3.2 —	0.3 +	0.9 +	0.1	— 0.952	(Δh_3
					+	15.0 +	2.6 —	2.2 +	0.2 —	1.0	0.0	— 0.109	(Δk_3
						+	156.4 --	70.0 —	0.5 —	0.4 —	2.0	— 2.705	(Δh_4
							+	39.1 +	0.2 +	0.1 +	0.9	+ 5.050	(Δk_4
								+	0.1	0.0	0.0	+ 0.096	(y_2
										+ 0.5	0.0	+ 0.227	(y_3
											0.0	+ 0.102	(y_4

ANGLES — GILL AND FINLAY.

x_1	Δh_1	Δk_1	x_2	Δh_2	Δk_2	x_3	Δh_3	Δk_3	Δh_4	Δk_4	y_2	y_3	y_4	n	
0.2 +	2.3 —	1.2 —	0.1 —	5.6 +	2.3 —	0.1 +	1.0 +	0.3 +	0.9 +	2.7 +	0.6 +	0.3	0.0	+ 3".597	(Δp_1
0.1 —	1.2 —	0.2 —	0.6 +	3.6	0.0 +	0.3 —	1.7 +	0.8 —	1.2 +	1.1 —	0.1 —	0.5	0.0	— 0.123	(Δq_1
2.8 —	6.6 +	1.0 —	3.3 +	11.3 —	0.3 —	5.3 +	3.9 —	5.2 —	0.9 —	0.6 —	0.9 —	0.1 —	0.1	+ 39.882	(Δp_2
0.7	0.0 +	0.7 +	1.7 —	0.2 —	2.3 —	0.2 —	3.2 +	1.8 —	0.2 +	0.3 —	0.1 —	0.6	0.0	+ 59.512	(Δq_2
0.2 +	1.7 —	0.5 —	3.1 —	2.1 +	2.4 —	8.0 —	2.5 —	5.3 +	2.8 +	3.5 +	0.2 —	1.3	0.0	+ 8.151	(Δp_3
1.1 +	0.7	0.0 —	1.4 —	1.8 +	1.9 —	14.1 +	2.8 —	13.3 +	4.2 +	1.2 +	0.3 —	0.2	0.0	— 149.849	(Δq_3
2.0 —	2.1 —	3.0 +	1.3 —	2.4 —	2.4 —	4.2 —	4.2 —	5.1 +	14.3 +	27.5 —	0.3 +	1.8 +	1.4	+ 345.750	(Δp_4
1.1 —	0.1 —	0.9 +	3.9 --	2.8 —	3.9 —	7.5 —	1.9 —	4.1 —	8.1 +	26.6 —	0.1 +	1.1 +	1.2	— 50.123	(Δq_4
4.8 +	74.4 —	32.0 —	52.2 —	52.7 +	61.4 —	18.4 —	98.0 +	132.7 +	320.2 —	342.4 —	1.2 +	17.3 +	13.7	— 19.419	(μ
5.6 +	8.6 —	8.4 —	14.3 —	16.1 +	15.7 +	12 5 +	1.1 +	19.8 +	22.6 —	21.9 +	1.0 —	3.5 —	2.5	— 0.047	(z_0
1.1 +	1.0 —	0.9 —	4.8 —	6.2 +	2.6 —	10.6 +	9.0 —	5.2 +	0.7 +	4.2 +	0.3	0.0 +	0.6	+ 4.685	(z_F
1.2 +	60.8 —	30.8 —	22.2 +	4.3 +	19.5 +	26.4 —	39.2 +	26.6 —	59.0 +	30.5 +	2.2 —	0.3 +	4.6	+ 17.937	($\Delta \varepsilon_1$
3.0 —	1.1 —	1.4 +	9.5 —	30.7 +	20.5 +	27.5 —	65.5 +	0.5 +	68.5 +	20.1 —	13.5 +	2.4	0.0	— 0 274	($\Delta \varepsilon_2$
2.1 —	57.5 +	1.6 —	11.4 —	42.0 +	8.4 —	139.0 +	204.8 —	158.4 —	187.5 +	83.3 +	1.9 —	23.7 —	14.3	— 47.711	($\Delta \varepsilon_3$
0.6 +	27.6 +	6.9 +	59.0 +	110.2 —	46.5 —	168.8 +	232.0 —	85.5 —	1053.9 +	817.0 —	9.2 —	5.8 +	1.1	— 148.940	($\Delta \varepsilon_4$
2.8 —	120.3 +	64.3 +	43.9 +	61.2 —	33.7 —	78.4 +	98.3 —	55.0 +	174.0 —	84.3 +	1.0 +	2.2 —	3.3	+ 16.221	(x_1
+	246.5 —	97.7 —	71.6 —	126.6 +	66.3 +	124.2 —	194.7 +	107.0 —	319.6 +	178.8 —	1.0 —	1.0 +	2.9	— 27.449	(Δh_1
		+ 64.2 +	26.9 +	53.0 —	28.8 —	58.4 +	86.2 —	54.5 +	141.1 —	72.8 +	1.1 +	1.1 —	3.6	+ 11.596	(Δk_1
			+ 260.0 +	307.6 —	212.5 +	115.0 —	171.8 +	80.8 —	331.8 +	167.2 +	0.1 +	1.9 —	1.2	— 24.745	(x_2
				+ 581.4 —	292.1 +	150.2 —	278.7 +	141.6 —	576.6 +	297.3 +	11.0 +	2.7 —	2.0	— 39.038	(Δh_2
				+ 209.8 —	91.1 +	160.9 —	80.1 +	306.8 —	164.6 —	1.7 —	1.5 +	0.2	+ 21.255	(Δk_2
					+ 767.2 —	953.8 +	560.7 +	625.6 —	320.4 +	1.0 —	9.8 —	2.7	— 46.123	(x_3
						+ 1653.8 —	823.8 —	1037.2 +	566.1 —	1.6 —	7.3 —	3.5	+ 73.033	(Δh_3
							+ 562.1 +	575.1 —	298.4 +	2.9 —	4.4 —	0.4	— 40.294	(Δk_3
								+ 5485.0 —	2694.6 —	14.5 +	5.0 —	11.5	+ 173.188	(Δh_4
									+ 1820.3 +	3.4 —	0.1 +	10.0	— 63.029	(Δk_4
										+ 3.6	0.0 —	0.3	— 0.690	(y_2
											+ 27.4 +	7.2	+ 1.782	(y_3
												+ 14.8	+ 0.181	(y_4

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From these normals two entirely independent solutions were made.

I. At the Cape a rigorous solution was made, under the direction of Mr. S. S. Hough, of the normals E. The absolute terms used in this solution were however slightly different from those given above for the following reasons:

a) The correction for differential aberration was not included.

b) Some mistakes, which I found by a thorough final revision of all the computations, had not yet been corrected. These errors were generally not of a very serious nature, and as a matter of fact their correction has caused only small alterations in the resulting values of the unknowns. In no case were the coefficients of the equations of condition affected.

c) Three observations [nrs 87, 283 and 467 [1])], which ought to have been rejected, had actually been included in the formation of the normal equations. As it was desirable that the left hand members of the normal equations should not be changed, the values of the right hand members of the equations of condition resulting from the substitution of the Cape solution were adopted instead of the actual (O—C)'s and the effect of this alteration on the absolute terms of the normal equations was computed. As the final solution differs very little from the Cape solution, these equations will have a residual practically equal to zero after substitution of the final values of the unknowns, and thus the fact that they were not rejected will not influence the values of the unknowns, but only have a (very small) effect on the weights.

Two different solutions of the normals E were made at the Cape:

α) The rigorous conditions

(1) $$\Delta\varepsilon_1 - 3.1\varepsilon_2 + 2.1\varepsilon_3 = 0$$

(2) $$y_2 = 1.468\, y_4$$

were *not* introduced. The values of $\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_2$, y_2 and y_4 alone were derived from this solution, with their respective weights.

β) The conditions (1) and (2) were introduced into the normals and all the unknowns were solved with their weights. The differences between the values of $\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_3$, y_2 and y_4 from this solution and those from solution **α** were of the same order of magnitude as the probable errors.

II. Another solution was made by myself. This was based on the normals C and D, in which the conditions (1) and (2) were *not* introduced. The leading principle of this solution was the following. In the normals D, which are derived from the position-angles, the coefficients of the unknowns μ, z, $\Delta\varepsilon$, x, Δh, Δk, y are very small compared to those of the p's and q's. Consequently a very satisfactory first approximation for these last unknowns may be derived from the normals D in which all the other unknowns are neglected. Then these values of p and q are substituted in the normals C, and the unknowns μ, z, $\Delta\varepsilon$, x, Δh, Δk, y are solved. The resulting values are again substituted in the normals D, and a second approximation for the values of p and q is derived, which is then again substituted in the normals C, and so on.

For this purpose a general solution was made of the two sets of normals, giving the values of the unknowns in terms of the right hand members of the normal equations. Consequently the whole process of successive approximation is reduced to substituting the successive values of these right hand members in the general solution. A symmetrical arrangement of the computations of such a general solution, which has proved very convenient in practice, will be explained in the appendix.

In this solution the same erroneous absolute terms, on which the Cape solution was based, were first used. The agreement between my results and those derived by the Cape computers proved to be so

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[1]) In the case of 467 a different pair of satellites was observed in the two positions of the semilenses (see page 7). Nrs 87 and 283 gave residuals of +0".974 and −0".880 (in the Cape solution). In both cases the discordances are almost exactly equal to 30' of position-angle, so that a mistake is very probable, although there is no evidence for it from the comparison of the individual pointings.

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satisfactory, that I felt no hesitation in using only my own method of solution to apply the corrections mentioned above sub a, b and c, without going through the additional labour of also correcting the Cape solution. As the left hand members of the normals were not affected by these corrections, the same general solutions could be used, only the values of the absolute terms, which must be substituted in them, being altered.

The normals as given above were somewhat transformed before they were used for the solution. In the normals D, in order to make the coefficients of all the unknowns of the same order of magnitude, I introduced $\frac{1}{3}\Delta p_1$, $\frac{1}{3}\Delta q_1$, $\frac{1}{2}\Delta p_2$ and $\frac{1}{2}\Delta q_2$ as unknowns instead of Δp_1, Δq_1, Δp_2 and Δq_2. These normals thus become:

Normals P.

$\frac{1}{3}\Delta p_1$	$\frac{1}{3}\Delta q_1$	$\frac{1}{2}\Delta p_2$	$\frac{1}{2}\Delta q_2$	Δp_3	Δq_3	Δp_4	Δq_4	
+ 2534	— 350	— 127	— 317	— 62	+ 59	— 91	— 108	$= 3P_1$
	+ 2207	+ 61	— 162	— 132	— 186	+ 110	+ 117	$= 3Q_1$
		+ 2367	— 161	— 85	+ 177	+ 86	— 260	$= 2P_2$
			+ 3135	+ 85	— 82	— 203	+ 130	$= 2Q_2$
				+ 1484	+ 21	+ 138	— 186	$= P_3$
					+ 1927	— 301	+ 76	$= Q_3$
						+ 5106	+ 512	$= P_4$
							+ 5604	$= Q_4$

of which the solution is:

Solution P.

$3P_1$	$3Q_1$	$2P_2$	$2Q_2$	P_3	Q_3	P_4	Q_4	
+ 412	+ 68	+ 25	+ 46	+ 23	— 6	+ 5	+ 7	$= \frac{1}{3}\Delta p_1$
	+ 474	— 9	+ 30	+ 42	+ 45	— 5	— 8	$= \frac{1}{3}\Delta q_1$
		+ 432	+ 20	+ 28	— 43	— 12	+ 23	$= \frac{1}{2}\Delta p_2$
			+ 328	— 16	+ 16	+ 15	— 8	$= \frac{1}{2}\Delta q_2$
				+ 686	— 12	— 23	+ 26	$= \Delta p_3$
					+ 534	+ 34	— 14	$= \Delta q_3$
						+ 201	— 21	$= \Delta p_4$
							+ 183	$= \Delta q_4$

where the coefficients are given in units of the *sixth* decimal place, and where:

	n	$\Delta\varepsilon_4$	Δh_4	Δk_4	$\frac{1}{2}x_3$	$\frac{1}{2}\Delta k_3$	Δh_3	$\frac{1}{3}x_2$	$\frac{1}{2}\Delta h_2$	$\frac{1}{3}\Delta k_2$	$\frac{1}{5}x_1$	$\frac{1}{3}\Delta h_1$	$\frac{1}{5}\Delta k_1$	$\frac{1}{2}\Delta\varepsilon_2$	$\frac{1}{3}\Delta\varepsilon_1$	$\Delta\varepsilon_3$	y_2	y_3	y_4
$3P_1 =$	+ 11".35	— 13	— 24	— 3	+ 16	+ 11	— 2	+ 6	+ 51	— 39	— 23	— 23	+ 20	— 73	+ 58	+ 4	— 3	— 2	0
$3Q_1 =$	— 12.98	— 9	+ 93	— 42	— 19	— 25	+ 7	— 3	— 35	0	+ 34	— 14	+ 58	— 12	— 78	— 5	+ 1	+ 2	— 1
$2P_2 =$	+ 78.49	+ 48	— 4	+ 4	+ 26	+ 29	— 11	— 17	— 81	+ 48	— 17	+ 72	— 42	+ 75	— 72	— 45	+ 3	— 3	0
$2Q_2 =$	+ 112.63	— 33	+ 29	— 17	— 27	+ 34	+ 36	— 41	— 56	+ 55	— 9	— 4	— 14	— 73	— 28	— 8	— 1	+ 1	0
$P_3 =$	+ 8.37	+ 7	+ 6	— 7	+ 10	+ 22	+ 3	— 27	— 30	+ 27	— 6	+ 18	— 12	— 33	+ 3	+ 41	0	— 6	0
$Q_3 =$	— 148.09	— 31	— 8	+ 3	+ 46	+ 59	— 19	— 3	+ 10	— 1	— 9	+ 18	— 10	— 22	+ 7	— 71	0	— 4	0
$P_4 =$	+ 327.87	+ 75	+ 196	— 126	+ 32	+ 33	— 5	0	— 11	+ 18	+ 82	— 67	+ 97	+ 32	— 15	+ 4	— 1	— 5	— 5
$Q_4 =$	— 58.50	— 188	+ 113	— 86	+ 43	+ 31	— 14	— 13	— 1	+ 19	+ 31	— 28	+ 33	— 1	— 19	— 17	0	— 3	— 2

In the normals C $\frac{1}{7}z_G$, $\frac{1}{10}z_F$, $\frac{1}{3}\Delta\varepsilon_1$, $\frac{1}{2}\Delta\varepsilon_2$, $\frac{1}{5}x_1$, $\frac{1}{3}\Delta h_1$, $\frac{1}{5}\Delta k_1$, $\frac{1}{3}x_2$, $\frac{1}{2}\Delta h_2$, $\frac{1}{3}\Delta k_2$, $\frac{1}{2}x_3$ and $\frac{1}{2}\Delta k_3$ were introduced as unknowns instead of z_G, z_F, $\Delta\varepsilon_1$ .... etc., and the order of the unknowns was altered for the convenience of the solution (*viz.* so that no „multiplicators" [see appendix] larger then unity should appear). The unknowns y_2, y_3 and y_4 were neglected in first approximation. I thus derived the following normal equations:

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Normals S.

$\Delta\varepsilon_4$	Δh_4	Δk_4	$\frac{1}{2}x_3$	$\frac{1}{2}\Delta k_3$	Δh_3	$\frac{1}{3}x_2$	$\frac{1}{2}\Delta h_2$	$\frac{1}{3}\Delta k_2$	$\frac{1}{5}x_1$	$\frac{1}{3}\Delta h_1$	$\frac{1}{5}\Delta k_1$	$\frac{1}{2}\Delta\varepsilon_2$	$\frac{1}{3}\Delta\varepsilon_1$	$\Delta\varepsilon_3$	μ	$\frac{1}{7}z_G$	$\frac{1}{10}z_F$	
+ 5756	— 1025	+ 792	— 336	— 168	+ 231	+ 182	+ 224	— 147	+ 11	+ 72	+ 66	— 101	+ 24	— 43	- 215	— 5	— 342	$=E_4$
	+ 5329	— 2625	+ 1245	+ 1145	— 1030	— 987	— 1141	+ 910	+ 817	— 911	+ 630	+ 130	— 153	— 186	+ 320	+ 158	+ 7	$=H_4$
		+ 1781	— 635	— 592	+ 563	+ 497	+ 590	488	— 396	+ 516	— 327	+ 42	+ 78	+ 82	— 343	— 154	+ 42	$=K_4$
			+ 3034	+ 2201	— 1885	+ 679	+ 586	— 532	— 782	+ 740	— 580	+ 102	+ 155	— 279	— 37	+ 175	— 211	$=2X_3$
				+ 2188	1613	+ 472	+ 547	— 462	— 551	+ 630	— 538	— 8	+ 155	— 325	+ 265	+ 276	— 103	$=2K_3$
					+ 1623	— 505	— 542	+ 469	+ 493	— 573	+ 427	— 119	— 116	+ 219	— 98	+ 8	+ 90	$=H_3$
						+ 2273	+ 1781	— 1866	+ 593	— 604	+ 356	— 5	— 246	— 41	— 157	— 301	— 143	$=3X_2$
							+ 2219	— 1693	+ 549	— 691	+ 472	— 172	— 48	— 86	— 105	— 225	— 123	$=2H_2$
								+ 1842	— 469	+ 562	— 400	+ 154	+ 212	+ 31	+ 184	+ 329	+ 77	$=3K_2$
									+ 2205	— 1732	+ 1510	+ 4	— 516	+ 7	— 224	— 197	— 54	$=5X_1$
										+ 2133	— 1386	+ 27	+ 594	— 184	+ 223	+ 181	+ 31	$=3H_1$
											+ 1495	— 65	— 498	+ 11	— 160	— 295	— 44	$=5K_1$
												+ 2300	— 165	+ 45	— 74	— 173	— 100	$=2E_2$
													+ 2050	— 106	+ 250	+ 130	— 16	$=3E_1$
														+ 1702	+ 8	+ 26	— 109	$=E_3$
															+ 2995	+ 876	+ 300	$=M$
																+ 1744	—	$=7Z$
																	+ 1053	$=10Z_F$

of which the solution is:

Solution S.

E_4	H_4	K_4	$2X_3$	$2K_3$	H_3	$3X_2$	$2H_2$	$3K_2$	$5X_1$	$3H_1$	$5K_1$	$2E_2$	$3E_1$	E_3	M	$7Z_G$	$10Z_F$	
+ 20	+ 2	— 18	+ 7	— 9	+ 12	— 7	+ 10	— 9	+ 6	+ 10	— 10	+ 2	— 2	+ 2	0	— 1	+ 8	$=\Delta\varepsilon_4$
	+ 124	+ 66	— 7	— 34	+ 96	+ 20	+ 98	— 4	— 10	+ 95	— 23	+ 2	— 15	+ 5	— 4	+ 5	— 4	$=\Delta h_4$
		+ 270	— 26	+ 22	— 129	+ 36	— 88	+ 57	— 37	— 111	+ 41	— 22	+ 12	— 7	+ 19	+ 4	— 24	$=\Delta k_4$
			+ 206	— 128	+ 123	— 110	+ 35	— 88	+ 89	+ 26	— 82	+ 3	— 8	— 2	+ 22	— 17	+ 12	$=\frac{1}{2}x_3$
				+ 289	+ 64	+ 88	— 69	+ 90	— 70	— 58	+ 98	+ 3	+ 14	+ 13	— 15	— 18	— 1	$=\frac{1}{2}\Delta k_3$
					+ 539	— 39	+ 215	— 78	+ 50	+ 235	— 80	+ 32	— 32	+ 16	— 7	— 33	+ 18	$=\Delta h_3$
						+ 379	— 48	+ 341	— 130	— 15	+ 139	— 19	+ 14	+ 5	— 18	+ 10	+ 11	$=\frac{1}{3}x_2$
							+ 335	+ 63	+ 39	+ 203	— 79	+ 19	— 43	+ 15	— 13	— 5	+ 15	$=\frac{1}{2}\Delta h_2$
								+ 513	— 110	— 52	+ 131	— 26	+ 3	+ 7	— 13	— 11	+ 13	$=\frac{1}{3}\Delta k_2$
									+ 248	+ 96	— 187	+ 1	— 12	+ 7	+ 14	— 17	+ 3	$=\frac{1}{5}x_1$
										+ 352	0	+ 20	— 38	+ 25	— 19	+ 9	+ 16	$=\frac{1}{3}\Delta h_1$
											+ 342	0	+ 29	+ 8	— 17	+ 31	+ 6	$=\frac{1}{5}\Delta k_1$
												+ 49	+ 2	0	— 3	+ 5	+ 6	$=\frac{1}{2}\Delta\varepsilon_2$
													+ 61	+ 1	— 3	+ 2	+ 2	$=\frac{1}{3}\Delta\varepsilon_1$
														+ 64	— 3	— 2	+ 10	$=\Delta\varepsilon_3$
															+ 46	— 23	— 14	$=\mu$
																+ 79	+ 6	$=\frac{1}{7}z_G$
																	+ 107	$=\frac{1}{10}z_F$

where the coefficients are given in units of the *fifth* decimal place, and where:

	n	$\frac{1}{3}\Delta p_1$	$\frac{1}{3}\Delta q_1$	$\frac{1}{2}\Delta p_2$	$\frac{1}{2}\Delta q_2$	Δp_3	Δq_3	Δp_4	Δq_4	y_2	y_3	y_4
$E_4 =$	— 136".15	+ 12	+ 1	— 42	+ 27	— 10	+ 27	— 68	+ 58	+ 9	+ 5	0
$H_4 =$	+ 175.89	+ 21	— 90	+ 6	— 28	— 8	+ 4	— 211	— 105	+ 14	— 5	+ 10
$K_4 =$	— 68.08	— 5	+ 38	— 3	+ 16	+ 3	— 4	+ 98	+ 60	— 3	0	— 9
$2X_3 =$	— 92.35	— 15	+ 17	— 4	+ 28	+ 6	— 17	— 24	— 28	— 2	+ 19	+ 5
$2K_3 =$	— 80.37	— 12	+ 21	— 8	+ 27	— 11	— 32	— 23	— 23	— 5	+ 7	+ 1
$H_3 =$	+ 73.98	— 1	— 2	+ 3	— 30	0	+ 16	+ 10	+ 16	+ 1	+ 8	+ 4
$3X_2 =$	— 79.16	— 5	+ 8	+ 37	+ 31	+ 36	+ 7	— 4	+ 2	0	— 5	+ 4
$2H_2 =$	— 88.46	— 18	+ 13	+ 36	+ 56	+ 34	— 6	+ 15	+ 6	— 22	— 4	+ 4
$3K_2 =$	+ 70.72	+ 18	0	— 46	— 41	— 35	— 5	— 11	— 7	+ 5	+ 4	— 1
$5X_1 =$	+ 77.32	+ 20	— 32	+ 25	+ 3	+ 5	+ 4	— 72	— 25	— 5	— 12	+ 16
$3H_1 =$	— 76.44	+ 2	+ 26	— 32	+ 4	— 23	— 20	+ 73	+ 29	+ 4	+ 2	— 8
$5K_1 =$	+ 52.80	— 2	— 54	+ 32	+ 7	+ 15	+ 10	— 82	— 29	— 6	— 5	+ 17
$2E_2 =$	— 2.90	+ 37	+ 24	+ 2	+ 15	+ 35	+ 27	— 27	— 6	+ 28	— 3	0
$3E_1 =$	+ 47.73	+ 4	+ 5	+ 35	+ 26	— 1	— 1	+ 5	+ 13	— 7	0	— 13
$E_3 =$	— 41.36	— 9	+ 13	+ 37	+ 25	— 19	+ 18	— 3	+ 11	— 2	+ 24	+ 14
$M =$	— 19.42	— 9	— 4	— 13	— 12	— 23	— 13	— 72	— 29	+ 1	— 17	— 14
$7Z_G =$	— 0.33	— 21	+ 12	— 30	— 25	— 43	— 19	— 58	— 32	— 7	+ 25	+ 17
$10Z_F =$	+ 46.85	+ 5	— 19	+ 7	— 10	— 25	— 9	— 28	— 13	— 3	0	— 6

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Finally there remain the normals for the determination of y_2, y_3 and y_4, which were taken from the set C without alteration. We have thus:

Normals Y.

y_2	y_3	y_4	
+ 3.5	0.0 —	0.3	$= Y_2$
	+ 26.8 +	7.2	$= Y_3$
	+	14.7	$= Y_4$

Solution Y.

Y_2	Y_3	Y_4	
+ 0.287	— 0.002	+ 0.007	$= y_2$
	+ 0.043 —	0.021	$= y_3$
		+ 0.078	$= y_4$

where:

n	$\Delta\varepsilon_4$	Δh_4	Δk_4	$\frac{1}{2}x_3$	$\frac{1}{2}\Delta k_3$	Δh_3	$\frac{1}{3}x_2$	$\frac{1}{2}\Delta h_2$	$\frac{1}{3}\Delta k_2$	$\frac{1}{5}x_1$	$\frac{1}{3}\Delta h_1$	$\frac{1}{5}\Delta k_1$	$\frac{1}{2}\Delta\varepsilon_2$	$\frac{1}{3}\Delta\varepsilon_1$	$\Delta\varepsilon_3$	μ	$\frac{1}{7}z_G$	$\frac{1}{10}z_F$	$\frac{1}{3}\Delta p_1$	$\frac{1}{3}\Delta q_1$	$\frac{1}{2}\Delta p_2$	$\frac{1}{2}\Delta q_2$	Δp_3	Δq_3	Δp_4	Δq_4
$Y_2 =$	−0.79	+9	+14	−3	−2	−5	+1	0	−22	+5	−5	+4	−7	+28	−7	−2	+1	−7	−3	+1	−1	−1	+1	0	0	+2	0
$Y_3 =$	+1.56	+5	−5	0	+19	+7	+8	−5	−4	+4	−12	+2	−5	−3	0	+24	−17	+25	0	+1	0	+3	0	+8	+4	+3	+2
$Y_4 =$	+0.08	0	+10	−9	+5	+1	+4	−4	+4	−1	+16	−8	+17	0	−13	+14	−14	+17	−6	0	+1	0	0	0	0	+3	+1

For the final solution, with the correct absolute terms, I used as a first approximation the values of the unknowns resulting from my first solution, which agreed very nearly with those from the Cape solution. These values being substituted in the expressions for P_1, Q_1, P_2, ... E_4, H_4, K_4, X_3, ... Y_2, Y_3, Y_4, the resulting values of these quantities were substituted in the solutions **P**, **S** and **Y**. This gave a second approximation for the values of the unknowns, which were then again substituted in the expressions for P_1, Q_1, ... E_4, H_4, ... Y_2, ..., and these corrected values again in the solutions, and so on until a sufficient degree of approximation was reached [1]).

The final values of the unknowns are:

$$\begin{aligned}
\tfrac{1}{3}\Delta p_1 &= +0.013842 \\
\tfrac{1}{3}\Delta q_1 &= -0.008847 \\
\tfrac{1}{2}\Delta p_2 &= +0.036347 \\
\tfrac{1}{2}\Delta q_2 &= +0.042248 \\
\Delta p_3 &= -0.002761 \\
\Delta q_3 &= -0.069338 \\
\Delta p_4 &= +0.063842 \\
\Delta q_4 &= -0.013201
\end{aligned}$$

$$\begin{aligned}
\Delta\varepsilon_4 &= -0.02061 \\
\Delta h_4 &= +0.06259 \\
\Delta k_4 &= +0.02855 \\
\tfrac{1}{2}\ x_3 &= -0.00275 \\
\tfrac{1}{2}\Delta k_3 &= -0.02536 \\
\Delta h_3 &= +0.05916 \\
\tfrac{1}{3}\ x_2 &= -0.00678 \\
\tfrac{1}{2}\Delta h_2 &= +0.00594 \\
\tfrac{1}{3}\Delta k_2 &= -0.01817 \\
\tfrac{1}{5}\ x_1 &= +0.01613 \\
\tfrac{1}{3}\Delta h_1 &= +0.00777 \\
\tfrac{1}{5}\Delta k_1 &= -0.01560 \\
\tfrac{1}{2}\Delta\varepsilon_2 &= -0.00039 \\
\tfrac{1}{3}\Delta\varepsilon_1 &= +0.03399 \\
\Delta\varepsilon_3 &= -0.02583 \\
\mu &= +0.01583 \\
\tfrac{1}{7}\ z_G &= +0.00620 \\
\tfrac{1}{10}\ z_F &= +0.03114
\end{aligned}$$

$$\begin{aligned}
y_2 &= -0.1153 \\
y_3 &= +00.444 \\
y_4 &= -0.0172
\end{aligned}$$

1) Theoretically this is what was done, though actually I did not derive successive *approximations* to the values of the unknowns, but successive *corrections* to the first approximation. Besides the convenience of a rapidly decreasing number of significant figures, this method of successive corrections has the advantage that at the same time the residuals, which remain after the substitution of each approximation in the normal equations, are taken into account in the next approximation, in the manner which is explained in the appendix.

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Returning to the original unknowns we have:

$\Delta p_1 = +\ 0.04153$	weight 276	p. e.	$\pm$ 0.00523
$\Delta q_1 = -\ 0.02654$	245		0.00557
$\Delta p_2 = +\ 0.07269$	594		0.00356
$\Delta q_2 = +\ 0.08450$	770		0.00328
$\Delta p_3 = -\ 0.00276$	1482		0.00226
$\Delta q_3 = -\ 0.06934$	1889		0.00200
$\Delta p_4 = +\ 0.06384$	5115		0.00123
$\Delta q_4 = -\ 0.01320$	5483		0.00118
$\mu = -\ 0.01583$	2133		$\pm$ 0.00188
$z_F = +\ 0.0434$	26		0.0170
$z_G = +\ 0.3114$	9		0.0289
$\Delta\varepsilon_1 = +\ 0.10197$	194		$\pm$ 0.00625
$\Delta\varepsilon_2 = -\ 0.00078$	503		0.00388
$\Delta\varepsilon_3 = -\ 0.02583$	1568		0.00219
$\Delta\varepsilon_4 = -\ 0.02061$	5205		0.00121
$\Delta h_1 = +\ 0.0233$	34		$\pm$ 0.0150
$\Delta k_1 = -\ 0.0780$	13		0.0241
$\Delta h_2 = +\ 0.0119$	79		0.0097
$\Delta k_2 = -\ 0.0545$	23		0.0181
$\Delta h_3 = +\ 0.05916$	195		0.00620
$\Delta k_3 = -\ 0.05072$	90		0.00912
$\Delta h_4 = +\ 0.06259$	826		0.00302
$\Delta k_4 = +\ 0.02855$	389		0.00441
$x_1 = +\ 0.0807$	17		$\pm$ 0.0211
$x_2 = -\ 0.0203$	31		0.0156
$x_3 = -\ 0.00550$	123		0.00783
$y_2 = -\ 0.1153$	3		$\pm$ 0.0502
$y_3 = +\ 0.0444$	23		0.0181
$y_4 = -\ 0.0172$	12		0.0251

The weights derived from the Cape solution have been adopted, those of $\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_3$, y_2 and y_4 being taken from the solution α, in which no rigorous conditions were introduced.

Substituting the adopted values of the unknowns in the individual equations of condition, we get the residuals v which are given in the last columns of Tables IV—XV. From these residuals we derive (all observations together).

$$[pvv] = 6.2394$$

$$\textit{p. e. of observation of weight unity } \varrho = \sqrt{\frac{[pvv]}{n-m}} = \pm\, 0''.0869$$

where n = number of observations = 405
m = number of unknowns = 29

From the distances alone we get:

$$[pvv] = 2.6958 \qquad \varrho = \pm\, 0''.0832.$$
$$n = 206 \qquad m = 29$$

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From the position-angles alone we find:

$$[pvv] = 3.5436 \qquad \varrho = \pm 0''.0965$$
$$n = 199 \qquad m = 26$$

For GILL alone (all observations together) we have

$$[pvv] = 4.8784 \qquad \varrho = \pm 0''.0867$$
$$n = 323 \qquad m = 28$$

FINLAY's observations are too few in number to derive a trustworthy probable error, but the fact that the p. e. derived from GILL's observations alone agrees exactly with that found from all observations, shews that the assumption wich was made in the beginning, *viz.* that the observations by the two observers are entitled to the same weight, must be very nearly correct.

As a check on the above values, the probable error was also derived from the sum of the residuals, all taken with the same sign (and each multiplied by the square root of the weight), by the formula

$$\varrho = 0.845 \frac{[(v)\sqrt{p}]}{\sqrt{n(n-m)}}$$

where (v) denotes the absolute value of the residual v.

I find in this way:

All observations:	$[(v)\sqrt{p}]$ =	38.220	$\varrho = \pm$ 0''.0828
Distances :		18.550	0''.0820
Position-angles :		19.670	0''.0896
GILL *alone* :		30.318	0''.0830

The agreement with the first values is very satisfactory. It is well known that this method generally gives a smaller probable error, which is explained by the fact, that in practice we nearly always find a somewhat larger number of large residuals than according to the theory, and these large residuals have more influence on the sum of the squares than on the simple sum. We must expect a priori that such large residuals, which generally are due to mistakes, are more frequent in the position-angles than in the distances. In observing position-angles, the microscope readings are read out to an assistant, and recorded by him, in the distance observations they are printed automatically, so that mistakes are practically excluded. Accordingly the probable errors derived from the distances alone by the two methodes are practically identical, while in the case of the position-angles there is a sensible difference. We can thus safely assume that the larger probable error of the position-angles is, at least for a great part, due to these exceptionally large residuals, and that the true accidental errors of observation are practically the same for the two kinds of observations. I have adopted as the probable error of an observation of weight unity that found from the discussion of all observations together *viz:*

$$\varrho = \pm 0''.0869.$$

In this connection it is interesting to compare the actual numbers of residuals between certain limits, with the same number as given by the theory of probability of errors. Adopting for the probable error of the observation of weight unity $\varrho = \pm 0''.0869$, I find the following numbers of residuals (each multiplied by the square root of the weight):

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Residual	DISTANCES		POSITION ANGLES	
	Actual number	Theoretical number	Actual number	Theoretical number
0".00—0".05	74	62.2	65½	60.1
0.05 —0.10	52½	53.6	59	51.7
0.10 —0.15	42½	40.0	33½	38.6
0.15 —0.20	18	25.3	17½	24.5
0.20 —0.30	17	20.8	16½	20.1
> 0.30	2	4 1	7	4.0
	206	206.0	199	199.0

Residuals, which are exactly equal to one of the limits of two groups were counted as ½ in each of the groups. The conclusion arrived at above, that for the position-angles the number of large residuals must slightly exceed the theoretical number, is confirmed. On the whole the agreement between the actual and theoretical numbers is very satisfactory.

A very important question is, whether after correction for $\varkappa$, or z, the observations of distance are really free from systematic errors. To investigate this question the observations were arranged in order of distance, and for every *five* consecutive distances the mean residual was taken (no weights being applied). The observations of half and double distances, and the observations in which the amount of crossing was measured, were treated separately. In the following table these mean distances and mean residuals are given in order of distance, the means which depend on observations of the two just mentioned exceptional kinds are distinguished by a prefixed letter.

TABLE XVIB. MEAN RESIDUALS. — DISTANCES

GILL.						FINLAY.	
Mean distance.	Mean residual.	Mean distance.	Mean residual.	Mean distance.	Mean residual.	Mean distance.	Mean residual.
"	"	"	"	"	"	"	"
d,h 8	— 0.182	197	— 0.019	417	+ 0.007	ψ 79	— 0.026
ψ 26	— 0.025	217	+ 0.018	437	+ 0.003	97	— 0.013
ψ 38	— 0.045	227	— 0.013	456	+ 0.024	165	— 0.101
ψ 49	+ 0.051	260	— 0.033	504	+ 0.016	218	+ 0.063
63	— 0.051	275	— 0.040	542	— 0.078	274	— 0.010
ψ 81 †	— 0.094	291	— 0.010	567	— 0.005	401	— 0.028
103	+ 0.127	305	— 0.086	622	— 0.056	474 *	— 0.078
133	— 0.026	322	— 0.004	686	— 0.051	578 *	+ 0.031
148	+ 0.065	343	+ 0.072	785	+ 0.079	722 *	+ 0.088
170	— 0.117	368	+ 0.077	871 *	+ 0.035		
183	+ 0.062	394	+ 0.024	968 *	+ 0.009		

d,h Observations of double and half distances.
ψ Observations in which ψ was recorded.
† *six* observations. * *four* observations.

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These residuals show no sign of any systematic error depending on the distance. The first value may seem to show that the distances measured by the method of half or double distances are systematically measured too short, but it is very doubtful whether any legitimate conclusion can be based on the mean of only five observations.

If there was a systematic error proportional to the distance, and *common to both observers*, the amount of this error would be included in the determination of μ, the coefficient of which is proportional to s. But such an error is eliminated a priori by the use of the standard stars for the determination of the scale-value, the true distance of which is known within $\frac{1}{60000}$ part of its amount (see first Chapter page 8). A systematic error which follows any other law, whether common to both observers or not, would of necessity appear in the residuals of Table XVIA. It can thus be asserted with great confidence that, *after correction for $\varkappa$, the heliometer-observations of distances up to 1000" made by* GILL *and* FINLAY, *are free from systematic error*, provided the scale-value be determined by the observation of standard stars of which the distance is well known [1]).

It will be remembered that z_G and z_F are the corrections on account of „crossing through" to observations by each of the observers of a distance of 100", or

$$z = \frac{\varkappa^2}{200}$$

From the values of z_G and z_F which were found by the solution of the equations, we derive

$$\varkappa_G = 2".95 \qquad \varkappa_F = 7".89$$

From the distances in which $\varkappa$ was directly observed we found (Chapter I, Table II, page 10)

$$\varkappa_G = 3".04 \qquad \varkappa_F = 6".67$$

The agreement is practically absolute, having regard to the small number of observations, on which FINLAY's result rests. We can thus safely assert that, at least for GILL and FINLAY, the systematic correction inversely proportional to the distance is not a purely empirical result, but represents a true physical fact, which is a necessary consequence of the method of observing with the modern heliometer.

For the position-angles the assumption was made (Chapter I, page 13) that they are not affected by any systematic error. It was pointed out there that the only kind of error, which would be of importance for the present work, would be one depending on the distance. To see whether any such error does exist I again arranged the observations in order of distance and took the mean of every *five* consecutive residuals. The observations by the two observers were treated seperately; the means are given in table XVIB, arranged in order of distance, those depending on FINLAY's observations being distinguished by a prefixed F. It will be seen that there is no evidence of any systematic error depending on the distance.

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1) It has been proved by Sir DAVID GILL (Cape Annals, Vol VI, Part 1, page 146), that there are no sensible systematic errors in the heliometer observations of distance depending on the position-angle.

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TABLE XVIB. MEAN RESIDUALS. — POSITION-ANGLES.

Mean distance.	Mean residual.	Mean distance.	Mean residual.	Mean distance.	Mean residual.	Mean distance.	Mean residual.
"	"	"	"	"	"	"	"
8	+ 0.025	157	— 0.019	282	— 0.035	494	— 0.031
26	— 0.191	F 165	— 0.055	297	+ 0.064	F 507	— 0.122
39	— 0.023	175	+ 0.055	312	— 0.050	531	+ 0.017
47	— 0.049	190	— 0.012	331	+ 0.074	557	+ 0.083
62	— 0.031	204	+ 0.124	363	+ 0.015	605	+ 0.047
F 65	+ 0.085	F 218	— 0.024	385	— 0.010	655	— 0.072
86	— 0.105	220	+ 0.013	406	+ 0.072	F 684 †	— 0.163
109	— 0.004	240	+ 0.029	F 413	— 0.004	746	— 0.025
F 119	— 0.059	265	— 0.017	432	+ 0.007	823 *	+ 0.102
140	— 0.042	F 274	— 0.185	450	— 0.016	942 *	+ 0.143

F Observer Finlay.
† *six* observations. * *four* observations.

From the conclusion arrived at just now, *viz.* that the observations of distance are free from systematic error, it follows further that the reciprocal of the mass of Jupiter's system, which is determined exclusively from the distances, is also free from systematic error, except in so far as it is affected by the remaining uncertainty in the adopted distance of the standard stars. This uncertainty corresponds to a probable error (see Chapter I, page 8) of:

$$\pm 3 \frac{1047.7}{6705.4} \times 0.065 = \pm 0.030$$

On the other hand the probable error resulting from the errors of observation is:

$$\pm 3 \frac{1047.7}{100} \times 0.0019 = \pm 0.060.$$

The resulting true probable error is ± 0.067

The correction to the adopted reciprocal of the mass is

$$\Delta \odot M = \frac{3 \odot M}{100} \mu = - 0.498.$$

We have seen (Chapter I, page 7), that the fact that the observations were not corrected for parallax introduces a slight systematic error in the distances, depending on the hour-angle. The hour-angle has always been between ± 1^h and ± 4^h. If it is desired to take this error into account it will be sufficiently accurate to adopt a constant correction $\frac{\delta s}{s} = - 0.00000\,75$, which gives

$$\delta \odot M = + 0.024.$$

The final value of the reciprocal of the mass of Jupiter's system, derived from these observations is thus

$$\odot M = 1047.226 \pm 0.067.$$

NEWCOMB's value (*Astronomical constants*, page 97) is

$$\odot M = 1047.35 \pm 0.065.$$

The two determinations agree within the limits given by the probable errors. The mean of the two determinations, *viz.*

$$\odot M = 1047.288 \pm 0.046$$

or in round numbers

$$\odot M = \mathbf{1047.29 \pm 0.05}$$

will probably be nearer to the truth than either of the two.

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It must now be investigated in how far the other unknowns can be assumed to be determined free from systematic errors.

If there is a personality in the observer's calling out the moment of observation, or in the assistant's recording this moment from the chronometer, or if there is an error in the adopted longitude of the Cape Observatory, or in the adopted constant of aberration, the sum of all these effects will be a constant error τ in all the observed times. This will produce an error in the computed distance of

$$\tau \frac{ds}{dt} = \tau\left(n_1 \frac{\partial s}{\partial \varepsilon_1} + n_2 \frac{\partial s}{\partial \varepsilon_2}\right)$$

and similarly for the position-angles.

It is thus not possible to determine the longitudes at the epoch free from systematic error, but what we can get from the observations is

$$\Delta\varepsilon_1 - n_1\tau \quad \Delta\varepsilon_2 - n_2\tau \quad \Delta\varepsilon_3 - n_3\tau \quad \Delta\varepsilon_4 - n_4\tau$$

and the other unknowns are not affected by this source of error. The error τ is unknown, and it is impossible to determine it. If we estimate its probable amount as $\pm 0^s.5$, which is almost certainly very much too large, the errors introduced in the unknowns

$$\Delta\varepsilon_1 \qquad \Delta\varepsilon_2 \qquad \Delta\varepsilon_3 \qquad \Delta\varepsilon_4$$

will be

$$\pm\ 0^\circ.00118 \quad \pm\ 0^\circ.00058 \quad \pm\ 0^\circ.00029 \quad \pm\ 0^\circ.0012.$$

The values of the unknowns with their true probable errors are thus:

(A)
$$\begin{aligned} \Delta\varepsilon_1 &= +0^\circ.1020 & \pm\ 0^\circ.0064 \\ \Delta\varepsilon_2 &= -\ 0.0008 & 0.00\ 39 \\ \Delta\varepsilon_3 &= -\ 0.0258 & 0.00\ 22 \\ \Delta\varepsilon_4 &= -\ 0.0206 & 0.00\ 12. \end{aligned}$$

It may be remarked that this systematic error disappears from the condition

(1) $$\Delta\varepsilon_1 - 3\Delta\varepsilon_2 + 2\Delta\varepsilon_3 = 0.$$

From the values of the unknowns we find

$$\Delta\varepsilon_1 - 3\Delta\varepsilon_2 + 2\Delta\varepsilon_3 = +0^\circ.0527 \pm 0^\circ.0138.$$

The probable error is computed by the formula for a linear function of the unknowns [1]). This quantity is nearly four times its probable error, so that most probably it is real, and the inequality which is called *libration* by Laplace is not, as Delambre found it to be, entirely insensible [2]).

It may however be interesting to see what values of the unknowns would have been found, if we had adopted Delambre's result, *viz.* that the equation (1) is true for all times. These values can be easily derived by the following consideration [3]).

If instead of E_1, E_2, E_3 we write $E_1 - \lambda$, $E_2 + 3\lambda$, $E_3 - 2\lambda$ in the absolute terms of the normal equations, and then eliminate λ by means of the condition (1), it can easily be verified that we will get

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1) See *Chauvenet* II, pages 541—543. It is easy to shew that the formula (83) there given is equivalent to the following rule: *to find the mean error of a function*

$$F = Ax + By + Cz + \ldots\ldots,$$

substitute A, B, C... *for* [*an*], [*bn*], [*cn*]... *in the absolute terms of the normal equations. Let the solution of these altered normals be* $x_0\ y_0\ z_0 \ldots$, *then the required mean error will be given by the formula:*

$$\varepsilon_F^2 = \{Ax_0 + By_0 + Cz_0 + \ldots.\}\varepsilon^2,$$

where ε is the mean error of an observation of weight unity.

2) It may be remembered that Delambre actually found for the epoch 1750.0 $\vartheta = l_1 - 3l_2 + 2l_3 = 180^\circ.0176$ (Laplace, *Mécanique Céleste*, *t.* IV, *pag.* 136). The deviation from 180° was neglected by him, as being within the uncertainties of the determination.

3) I owe this simple method of showing the effect of introducing the rigorous condition (1) to Mr. Hough, who used it in the Cape solution.

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a symmetrical set of equations for the determination of $\Delta\varepsilon_2$ $\Delta\varepsilon_3$ and the other unknowns, which is identical to the set of normal equations, which would have resulted if $\Delta\varepsilon_1$ had been replaced by $3\Delta\varepsilon_2 - 2\Delta\varepsilon_3$ in the equations of condition prior to the formation of the normals.

If the elimination of λ is postponed to the end we get the values of $\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_3$ in terms of λ.

I find:

$$(\Delta\varepsilon_1) = \Delta\varepsilon_1 - 0.00529\ \lambda$$
$$(\Delta\varepsilon_2) = \Delta\varepsilon_2 + 0.00576\ \lambda$$
$$(\Delta\varepsilon_3) = \Delta\varepsilon_3 - 0.00131\ \lambda$$

where the quantities in brackets denote the values which would have resulted from a conditioned solution. If now we determine λ by the condition

$$(\Delta\varepsilon_1) - 3(\Delta\varepsilon_2) + 2(\Delta\varepsilon_3) = 0$$

we find

$$\lambda = +2.099$$

(B)
$$(\Delta\varepsilon_1) = +0^{\circ}.0913$$
$$(\Delta\varepsilon_2) = +0.0112$$
$$(\Delta\varepsilon_3) = -0.0284$$

The differences between these values and the values (A) derived from the unconditioned solution are too large to be naturally explained by the probable errors; and it seems better to adopt the values (A).

We now come to the unknowns x, Δh, Δk. These too depend almost exclusively on the observations of distance, and are therefore not affected by any systematic error, at least in so far as such a thing can ever be asserted of any quantity determined by one instrument and one method only. It will be remarked that the weights of these unknowns are very small compared with those of the p's and q's, although the coefficients in the equations of condition are of the same order of magnitude. This is not accidental, but a consequence of the nature of the problem.

If we neglect the excentricities and inclinations, and if we suppose the earth to be in the fundamental plane, we have

$$v = l \qquad \rho = 1 \qquad B = 0 \qquad p - \mathrm{P} = \pm 90^{\circ}.$$

and the formulae (3) of page 19 become:

(3)
$$\frac{\partial s}{\partial v_1} = -\mathrm{K}_1 \cos(l_1 - L)$$
$$\frac{\partial s}{\partial \rho_1} = -\frac{\mathrm{K}_1}{\sin 1^{\circ}} \sin(l_1 - L)$$

where $\mathrm{K}_1 = \frac{a_1'' \rho_1}{\Delta_1'} \sin 1^{\circ} \sin(p - \mathrm{P})$

Further we have:

(4)
$$\frac{\partial s}{\partial h_1} = \mathrm{K}_1 \left[\cos l_1 \cos(l_1 - L) + \tfrac{1}{2} \sin l_1 \sin(l_1 - L)\right]$$
$$\frac{\partial s}{\partial k_1} = -\mathrm{K}_1 \left[\sin l_1 \cos(l_1 - L) - \tfrac{1}{2} \cos l_1 \sin(l_1 - L)\right]$$
$$\frac{\partial s}{\partial x_1} = -\mathrm{K}_1 \left[\sin \alpha_1 \cos(l_1 - L) - \cos \alpha_1 \sin(l_1 - L)\right]$$

and the same formulae hold for the other satellites, if the suffixes are altered, with the only difference that for satellite III γ is substituted for α in accordance with Marth's notation (see formulae II page 23) and for satellite IV only the first two equations must be considered.

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Now we have

$$\alpha_1 = 2(l_1 - l_2) = l_1 + c$$
$$\alpha_2 = l_1 - l_2 + 180° = l_2 + c + 180°$$
$$\gamma_3 = l_2 - l_3 + 180° = l_3 + c$$

where

$$c = l_1 - 2l_2 = l_2 - 2l_3 + 180°$$

Introducing these values, the equations (4) become after a simple transformation:

$$\frac{\partial s}{\partial h_i} = \tfrac{1}{2} \mathrm{K}_i \cos L \,[1 + \tfrac{1}{4} \cos (2l_i - L)]$$

(5)
$$\frac{\partial s}{\partial k_i} = -\tfrac{1}{2} \mathrm{K}_i \sin L \,[1 + \tfrac{1}{4} \sin (2l_i - L)]$$

$$\frac{\partial s}{\partial x_i} = \mp \tfrac{1}{2} \mathrm{K}_i \sin (L + c) \,[1 + \tfrac{1}{4} \sin (2l_i + c - L)]$$

where $i = 1, 2, 3$ or 4 and where the lower sign in the last formula must be taken for satellite II.

If we neglect the small periodic terms within the square brackets, the coefficients become

$$\frac{\partial s}{\partial h_i} = \tfrac{1}{2} \mathrm{K}_i \cos L$$

(6)
$$\frac{\partial s}{\partial k_i} = -\tfrac{1}{2} \mathrm{K}_i \sin L$$

$$\frac{\partial s}{\partial x_i} = \mp \tfrac{1}{2} \mathrm{K}_i \sin (L + c)$$

The angles L and c change very slowly, especially L, the variation during the time from 1891 Aug. 17 to Dec. 14 being for L: 7° and for c: 89°. If they were absolutely constant, it would evidently not be possible to determine the *three* unknowns Δh, Δk and x from the observations but only *one* linear function of them *viz:*

(7)
$$F_i = \cos L \, \Delta h_i - \sin L \, \Delta k_i \mp \sin (L + c) \, x_i$$

where again the lower sign must be taken for satellite II, and the last term must be dropped for satellite IV.

If we adopt for L and c their mean values, *viz:*

$$L = 205°.7 \qquad c = 11°.5$$

we find for the values of the functions F and their probable errors (computed by the rule page 67 footnote):

$$F_1 = -0.901 \, \Delta h_1 + 0.434 \, \Delta k_1 + 0.605 \, x_1 = -0°.1036 \pm 0°.0137$$
$$F_2 = -0.901 \, \Delta h_2 + 0.434 \, \Delta k_2 - 0.605 \, x_2 = -\ 0.0467 \pm\ 0.0086$$
$$F_3 = -0.901 \, \Delta h_3 + 0.434 \, \Delta k_3 + 0.605 \, x_3 = -\ 0.0720 \pm\ 0.0049$$
$$F_4 = -0.901 \, \Delta h_4 + 0.434 \, \Delta k_4 \qquad\qquad = -\ 0.0440 \pm\ 0.0027$$

The probable errors, though smaller than those of the separate values of Δh, Δk and x, are still very large. The reason of this must be sought in the circumstance that the satellites are not observed separately, but in pairs. If the same simplifications are introduced as before, the equations of condition become, as far as the quantities here considered are involved

(8)
$$\tfrac{1}{2}\mathrm{K}_1 F_1 - \tfrac{1}{2}\mathrm{K}_2 F_2 = \mathrm{O} - \mathrm{C}$$

If for the moment we introduce the new unknowns

$$f_i = \tfrac{1}{2} \frac{a_i''}{\Delta} \mathrm{F}_i \qquad (i = 1, 2, 3, 4)$$

and if in the expression for K_i we take

$$\rho_i = 1, \ \Delta_i' = \Delta, \qquad \sin (p - \mathrm{P}) = \pm 1.$$

the equation (8) becomes

$$f_1 - f_2 = \pm (\mathrm{O} - \mathrm{C})$$

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It is thus evident that in the simplified problem here considered we cannot determine the separate unknowns f_1, f_2, f_3, f_4, but only their differences. It is easy to see that for a similar reason it would not be possible to determine the unknowns Δh, Δk and x for each satellite separately, even if the angles L and c changed so much that the three unknowns referring to one satellite could be found separately.

In the actual problem however the neglected terms in formula (5) have some influence, and consequently it is possible to derive values for F_1, F_2, F_3 and F_4. Also the angles L and c do vary, and the values of Δh, Δk and x can be determined separately, but their determination is subject to relatively large uncertainties. It is one of the advantages of the method of least squares, that inherent uncertainties of this kind are automatically taken into account by a corresponding reduction of the weights. With the exception of Δh_1, Δh_2, x_2 and x_3 however even these separate values exceed their probable errors many times, and must thus be considered as real corrections. If they are added to the adopted values we get for the true values at Greenwich mean noon 1891 October 2:

$$h_1 = +\,0°.0203 \pm 0°.0150 \qquad k_1 = -\,0°.0746 \pm 0°.0241$$
$$h_2 = +\,0.0178 \pm 0.0097 \qquad k_2 = -\,0.0756 \pm 0.0181$$
$$h_3 = +\,0.0398 \pm 0.0062 \qquad k_3 = -\,0.2234 \pm 0.0091$$
$$h_4 = +\,0.5255 \pm 0.0030 \qquad k_4 = -\,0.6528 \pm 0.0044$$

From these values we derive the following instantaneous excentricities and perijovia (not to be confounded with the *excentricités et périjoves propres* of Laplace and Souillart).

$$E_1 = 0.000\,676 \pm 0.000\,205 \qquad \Omega_1 = 164°.8 \pm 11°.6$$
$$E_2 = 0.000\,678 \pm 0.000\,155 \qquad \Omega_2 = 166.75 \pm 7.65$$
$$E_3 = 0.001\,980 \pm 0.000\,079 \qquad \Omega_3 = 160.90 \pm 1.59$$
$$E_4 = 0.007\,313 \pm 0.000\,034 \qquad \Omega_4 = 141.166 \pm 0.249$$

(The probable errors are only roughly computed.)

The longitudes are reckoned in the fundamental plane from Jupiter's vernal equinox, of which the adopted longitude is 135°.3420 (Equinox and epoch 1891.75).

As to the x's, if we suppose that the corrections to the masses do not affect the terms of the second order in the large inequalities, we have:

$$\textit{from } x_1: \qquad \Delta m_2 = +\,0.00000405 \pm 0.00000106$$
$$\text{» } x_3: \qquad \Delta m_2 = -\,0.00000176 \pm 0.00000253$$
$$\text{» } x_2: \; 0.628\ \Delta m_1 + 1.092\ \Delta m_3 = -\,0.00000203 \pm 0.00000156$$

These values exceed their probable errors so little that no definite conclusions can be drawn from them. It appears probable however that the adopted mass of the second satellite requires a small positive correction.

From the values of y_2 and y_4 we find

$$y_2 - 1.468\ y_4 = -\,0.0900 \pm 0.0659.$$

Thus the rigorous condition (2) is satisfied as nearly as can be expected. The probable errors are however so large, that it is very doubtful whether the values found can be considered as real corrections. It would perhaps have been better to neglect these unknowns altogether. The representation of the equations of condition would not be sensibly altered by such a course. The corrections to the masses which correspond to the values of the unknowns are:

$$\textit{from } y_2: \quad \Delta m_3 = -\,0.0000694 \pm 0.0000302$$
$$\text{» } y_4: \quad \Delta m_3 = -\,0.0000152 \pm 0.0000222$$
$$\text{» } y_3: \quad \Delta m_4 = +\,0.0000188 \pm 0.0000077$$

Lastly we come to the unknowns Δp and Δq. These are determined almost exclusively from the position-angles. It has been shewn that the residuals give no evidence of any systematic error, we must

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thus here also consider the probable errors derived from the residuals as a true measure of the accuracy of the determination of the unknowns.

Adding the corrections found to the adopted values, we get

$$p_1 = +\,0''.0491 \pm 0''.0052 \qquad q_1 = -\,0''.0291 \pm 0''.0056$$
$$p_2 = +\,0.4889 \pm 0.0036 \qquad q_2 = -\,0.1769 \pm 0.0033$$
$$p_3 = +\,0.1138 \pm 0.0023 \qquad q_3 = -\,0.1390 \pm 0.0020$$
$$p_4 = +\,0.2332 \pm 0.0012 \qquad q_4 = +\,0.2896 \pm 0.0012$$

From these values we derive the following instantaneous inclinations and nodes:

$$i_1 = 0''.0571 \pm 0''.0053 \qquad \Omega_1 = 239°.35 \pm 5'.50$$
$$i_2 = 0.5199 \pm 0.0035 \qquad \Omega_2 = 250.108 \pm 0.367$$
$$i_3 = 0.1976 \pm 0.0021 \qquad \Omega_3 = 219.308 \pm 0.628$$
$$i_4 = 0.3718 \pm 0.0012 \qquad \Omega_4 = 321.157 \pm 0.186$$

It must be remembered that these are relatively to the adopted fundamental plane, which is defined by its adopted inclination and node in reference to a plane parallel to the earth's true equator.

It will be interesting to compute the probable errors of the computed places of the satellites at the epoch of the observations resulting from the probable errors which have been found for the corrections to the elements.

Referring to the differential formulae given in Chapter II and in this Chapter, we can easily derive the equations

$$\Delta x'' = \frac{a''}{\Delta} \sin 1° \left\{ \tfrac{1}{2} F + \cos (v - L)\, \Delta\varepsilon \right\} + \frac{x''}{3\odot''} \Delta \odot''$$

$$\Delta y'' = \frac{a''}{\Delta} \sin 1° \left\{ \sin v\, \Delta p - \cos v\, \Delta q \right\}$$

In these formulae the terms which are multiplied by a small factor, such as $\sin B$ or $\sin i$ have been neglected, and we have taken $\rho = 1$ and $\Delta' = \Delta$, which is sufficiently accurate for the present purpose.

In the formula for $\Delta y''$ we can consider the probable errors of Δp and Δq as indentical, we then get for the problable error of the computed coordinates y''

$$\varrho_{y''} = \frac{a''}{\Delta} \sin 1° \varrho_{p,q}$$

This formula gives the same numerical result for the four satellites, *viz.*

$$p.\ e.\ of\ computed\ y'' = \pm 0''.014$$

If in the formula for $\Delta x''$ we take for $\cos (v - L)$ its mean value $\frac{1}{2}\sqrt{2}$ and for x'' its maximum value $\frac{a''}{\Delta}$, we derive the following probable errors:

$$p.\ e.\ of\ computed\ x_1'' = \pm 0''.021$$
$$x_2'' \quad 0.021$$
$$x_3'' \quad 0.020$$
$$x_4'' \quad 0.021$$

It remains to be seen in how far these probable errors are representative of the real accuracy with which the coordinates can be computed, or in other words in how far the computed coordinates are *systematically* correct. For the coordinates x'' this has already been done by taking into account the probable error of the determination of $\odot''$. For the position-angles we must have regard to the uncertainty in the determination of the position-angle of the standard stars, which can introduce a common error in all the computed latitudes. We have seen (page 8) that this uncertainty corresponds to a probable error of $\pm 2''.1$. This is equivalent to the following quantities in the coordinates y'' of the different satellites

$$\pm 0''.001^5 \qquad \pm 0''.002^5 \qquad \pm 0''.004 \qquad \pm 0''.007$$

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These are therefore the probable amounts of the systematic errors to be feared in the computed coordinates y''.

In conclusion I give a *résumé* of the corrections found, which must be compared to the *résumé* of adopted constants given at the end of Chapter II (page 25).

Final value of reciprocal of mass of Jupiter's system.

From the Cape observations alone $\mathcal{M} = 1047.226 \pm 0.067$

Combined with NEWCOMB **1047.29 ± 0.05**

Corrections to elements and masses.

$$\Delta\varepsilon_1 = +\,0^\circ.1020 \pm 0^\circ.0064$$
$$\Delta\varepsilon_2 = -\,0.0008 \pm 0.0039$$
$$\Delta\varepsilon_3 = -\,0.0258 \pm 0.0022$$
$$\Delta\varepsilon_4 = -\,0.0206 \pm 0.0012$$

$$\vartheta = l_1 - 3l_2 + 2l_3 = 180^\circ.0527 \pm 0^\circ.0138$$

$$\Delta p_1 = +\,0^\circ.0415 \pm 0^\circ.0052 \qquad \Delta q_1 = -\,0^\circ.0265 \pm 0^\circ.0056$$
$$\Delta p_2 = +\,0.0727 \pm 0.0036 \qquad \Delta q_2 = +\,0.0845 \pm 0.0033$$
$$\Delta p_3 = -\,0.0028 \pm 0.0023 \qquad \Delta q_3 = -\,0.0693 \pm 0.0020$$
$$\Delta p_4 = +\,0.0638 \pm 0.0012 \qquad \Delta q_4 = -\,0.0132 \pm 0.0012$$

$$\Delta h_1 = +\,0^\circ.0233 \pm 0^\circ.0150 \qquad \Delta k_1 = -\,0^\circ.0780 \pm 0^\circ.0241$$
$$\Delta h_2 = +\,0.0119 \pm 0.0097 \qquad \Delta k_2 = -\,0.0545 \pm 0.0181$$
$$\Delta h_3 = +\,0.0592 \pm 0.0062 \qquad \Delta k_3 = -\,0.0507 \pm 0.0091$$
$$\Delta h_4 = +\,0.0626 \pm 0.0030 \qquad \Delta k_4 = +\,0.0285 \pm 0.0044$$

from x_1: $\Delta m_2 = +\,0.00000405 \pm 0.00000106$

„ x_2: $\Delta m_3 = -\,0.00000176 \pm 0.00000253$

„ x_3: $0.628\,\Delta m_1 + 1.092\,\Delta m_3 = -\,0.00000203 \pm 0.00000156$

from y_2: $\Delta m_3 = -\,0.0000694 \pm 0.0000302$

„ y_4: $\Delta m_3 = -\,0.0000152 \pm 0.0000222$

„ y_3: $\Delta m_4 = +\,0.0000188 \pm 0.0000077$

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CHAPTER IV.

It has been remarked in Chapter II that the results derived from the present series of observations are not sufficient by themselves for the determination of all the elements of the motions of Jupiter's satellites. Before entering on a discussion of the question, in how far these results need to be supported by other data, and what are the best means to derive the necessary supplementary data, a preliminary investigation of the relative accuracy with which the different unknowns of the problem are given by different kinds of observations appears desirable.

The most common method of determining the positions of the satellites, which has often been considered as the only accurate method, is by the observation of their eclipses.

About ancient eclipses DELAMBRE writes (*Astronomie théorique et pratique*, *Vol.* III, *page* 502): „Pour le premier satellite.... il n'est pas rare de voir deux observations d'une même éclipse différer entre elles d'une demi-minute; pour le second, la différence est plus que double; pour le troisième, elle peut aller jusqu'à 3^m; elle passe souvent 4^m pour le quatrième." It will be near enough for the present purpose if we adopt as the ***probable errors*** of the ancient observations one half of the quantities mentioned by Delambre This gives the following probable errors of one observation:

Ancient eclipses.

Satellite	In time	In arc (jovicentric)	Geocentric ($\log \Delta = 0.6$)
I	$\pm$ 15^s	$\pm$ 0°.0353	$\pm$ 0".090
II	35	0.0410	0.166
III	90	0.0524	0.339
IV	120	0.0300	0.340

To get similar data for modern eclipses, I compared all the observations of eclipses published in the *Astronomische Nachrichten* between 1890 and 1900, which have been observed by more than one observer [1]). This comparison gave the following probable errors:

Modern eclipses

I	$\pm$ 15^s	$\pm$ 0°.0353	$\pm$ 0".090
II	15	0.0176	0.071
III	20	0.0116	0.074
IV	40	0.0100	0.114

1) It is rather remarkable that of the observed phases more than three quarters of the whole number are emersions, not a single immersion either of I or II occurring. Can there be a connection between this fact and the circumstance that the emersions are visible after opposition, when Jupiter is easily observable in the evening, but the immersions before opposition, when the best time for observing is after midnight? At all events, whatever the reason may be, the fact is there, and the consequence is that the greater part of these observations is practically useless.

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Since 1878 the eclipses of the satellites have been regularly observed photometrically at the Harvard College Observatory. A number of photometric determinations of the light of the satellite, while it is undergoing an eclipse, is made; and from these the instant when the light is reduced to any definite proportion of the light of the uneclipsed satellite can be derived. Prof. R. A. Sampson, who has undertaken the discussion of these observations, has had the courtesy to communicate to me some of the results of a preliminary discussion of the observations of satellites III and IV. Prof. Sampson does not consider these results as representative of the precision which the method is capable of giving, and it is not yet possible to draw any definite conclusions as to the p. e. of one observation. It is however probable a priori, and I think I am justified in saying that this probability is so far fully confirmed by Prof. Sampson's results, that these probable errors will be many times smaller than those of the visual observations.

The probable error of a heliometer observation of weight unity has been found to be

$$\pm\ 0''.087.$$

This probable error is not directly comparable with those of eclipse observations, as the latter refer to one satellite only, while the heliometer gives the relative position of two satellites. We must thus, in order to compare the two kinds of observations, take into account the manner in which the different unknowns enter into the equations of condition given by each observation.

But first another kind of observations must be considered, which is directly comparable to the heliometer measures, *viz.* photography. Data about the accuracy of photographic measures of the satellites can be derived from the series of plates which was taken at Helsingfors in the years 1891—1895, and which have been measured and reduced by Mr. F. Renz (*Mémoires de l'académie de St. Pétersbourg*, VIII^me^ *série*, *Vol.* VII, No. 4.). On page 28 of his paper Mr. Renz gives the probable errors of the relative position of two satellites, derived from one „series" (Reihe), *i. e.* in the mean 5 images, both in R. A. and in declination. These probable errors vary between very narrow limits and are sensibly equal in both coordinates. The mean is

$$\pm\ 0''.046.$$

This is however the probable error of the measure on the plate, derived from the agreement *inter se* of the measures of the different images on the same plate. The analogon of this for the heliometer observations would be the p. e. derived from the agreement *inter se* of the four pointings of which one observation consists. The probable error of $\pm\ 0''.087$, which is derived from the discussion of the equations of condition, includes the effect of causes of error, which might be called „semisystematic", i. e. which remain constant for one observation, or for one night, such as the uncertainties of the determination of the instantaneous scale-value and index-error. In order that the photographic method shall be of equal value with the heliometer, the scale-value and especially the orientation of the plate must be determined with the same accuracy as the scale-value and index-error of the heliometer, and the freedom from systematic errors generally must be the same. It is not here the place to discuss the means by which this end can be attained. A more detailed discussion will be necessary to decide whether it has been attained in the case of the Helsingfors plates. I think however I am not overestimating the capabilities of the photographic method by expressing the conviction that sufficient means will be found to overcome all difficulties.

In that case one plate with three or four images will be equivalent to one complete heliometer observation (position-angle and distance). On a night when all six combinations are observed, each satellite is observed three times, and we have three independent determinations of each distance and each position-angle, such a night is therefore equivalent to three plates. If we reckon that three plates can be taken in a night, the same number of nights is necessary to get the same degree of precision by the two methods. The amount of time and labour involved in the taking of three photographic plates is however very much less than that which is necessary for six complete heliometer observations. As also the reductions

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will be simplified by the circumstance that the epoch is the same for the four satellites, the advantage from an economical point of vue is decidedly on the side of the photographic method.

We must now see how the different unknowns enter into the equations of condition given by observations of eclipses.

In the first place I will consider those unknowns which are determined from the longitudes and radii vectores, *viz.* the longitudes at the epoch, excentricities and perijovia, and the coefficients of the large inequalities. These are the unknowns which I have denoted by $\Delta\varepsilon$, Δh, Δk, and x.

If for the moment we leave the inequalities out of consideration, the correction to the longitude is

$$\Delta v = \Delta\varepsilon - \cos l\, \Delta h + \sin l\, \Delta k \tag{1}$$

Each eclipse observation gives an equation of condition

$$\Delta v = - n\, \Delta t$$

where Δt is the difference between the observed and predicted time of the eclipse. In this equation Δv must be replaced by its expression (1), in which Λ is subtituted for l (Λ = heliocentric longitude of Jupiter).

I will consider the ideal case of n eclipse observations regularly distributed over one complete revolution of Jupiter [1]) (*i. e.* 11 oppositions). Moreover I will suppose that an equal number of immersions and emersions have been secured, so distributed that the reduction of each observed phase to the middle of the eclipse can be effected with no more loss of weight than would be the case if both phases had been observed of every eclipse.

The number of epochs of true conjunction given by these observations will then be $\frac{1}{2}\, n$. It will easily be seen that these are the most favourable conditions for the determination of all the unknowns. We then get for the weights of $\Delta\varepsilon$, Δh and Δk respectively $\frac{1}{2}\, n$, $\frac{1}{4}\, n$ and $\frac{1}{4}\, n$.

For the unknowns x the conditions are entirely similar to those for Δh and Δk, only the observations need not be equally distributed over a complete revolution of Jupiter, but over a whole period of the inequality in question.

If we take $n = 50$ for each of the satellites, the whole number of observations will be about equal to the whole number of complete heliometer observations discussed in this paper. We will then have the following probable errors. (For photometric eclipses no definite probable errors can be given, but they will certainly be very much smaller than those of the modern visual observations).

	Satellite	*Eclipses*		*Heliometer*
		Ancient	*Modern*	
ε	I	± 0°.0071	± 0°.0071	± 0°.0062
	II	0.0082	0.0035	0.0039
	III	0.0105	0.0023	0.0022
	IV	0.0060	0.0020	0.0012
$h,\ k,\ x$	I	± 0°.0100	± 0°.0190	± 0°.0137
	II	0.0116	0.0050	0.0086
	III	0.0148	0.0033	0.0049
	IV	0.0085	0.0028	0.0027

We have seen in Chapter III that the determination of the unknowns h, k and x from the heliometer

1) or, which comes to the same thing, $\frac{1}{2}\, n$ with $\Lambda = 0°$ and $\frac{1}{2}\, n$ with $\Lambda = 90°$.

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observations is subject to particular uncertainties, owing to the fact that the satellites are observed in pairs, instead of individually as in the eclipses. The probable errors given for h, k ad x are those of the quantities F (page 69 formula (7)). As we have supposed the most favourable distribution that is possible for the eclipse observations, it is only fair at least to make a guess at the probable errors as they would result from heliometer observations in equally favourable conditions. It will be seen from the developments in Chapter III, that with a distribution of the observations over a whole period of Jupiter, or over two oppositions with an interval of three years (*i. e.* about 90° difference in the values of L), h, k and x will be determined with approximately the same p. e. which we now find for F.

A very important remark must be made with regard to this comparison of probable errors. We have supposed a number of 50 eclipse observations, taken at the most favourable epochs, and so that the reduction of the observed phase to the middle of the eclipse can be effected without loss of weight.

These ideal conditions will never be realised in practice [1]), and it is also very doubtful whether it is possible, at least for the outer satellites, to secure a series of as many as 50 eclipses, even in less favourable conditions, within a not unreasonably long time. The probable errors given for the values of the unknows derived from the eclipse observations must thus be considered as lower limits, which will never be reached in practice. On the other hand it is easy to secure as many heliometric or photographic observations as may be required, at any chosen epoch and in conditions which depend on the choice of the observer.

It must also be kept in mind that the heliometer observations have been shewn in Chapter III to be practically free from systematic errors. It is to be doubted whether the same thing can be said of eclipse observations. A possible source of systematic errors is mentioned by Laplace (*Mécanique céleste. t.* IV, *page* 136), which if it exists will affect the photometric measures in the same degree as the visual observations.

As a general, though necessarily somewhat vague, conclusion we can say that for the determination of those unknowns, which depend on longitudes and radii vectores, the ancient eclipse observations are very far inferior to the heliometer, and the modern (visual) eclipse observations can only give about the same accuracy if a very large number of eclipses is observed under such favourable conditions as will never be realised in practice. The photometric observations can reasonably be expected to give a higher precision than the heliometer.

We must now consider the latitudes, or the unknowns p and q. These are derived from the discussion of the durations of the eclipses.

This duration is given by the formula (*Tisserand IV*, *page* 76):

$$t' = 2T(1-X)\sqrt{\left\{1+\tfrac{1}{2}X+(1+x')\frac{s_o}{\beta}\right\}\left\{1+\tfrac{1}{2}X-(1+x')\frac{s_o}{\beta}\right\}}$$

from which follows

$$(2) \qquad (1+x')s_o = \frac{\beta\sqrt{4T^2(1-X)-t'^2}}{2T(1-X)}$$

where s_o = latitude referred to plane of Jupiter's orbit

$$T = \frac{\beta}{n-n'} \qquad \begin{cases} n = \text{mean motion of satellite} \\ n' = \text{" " " Jupiter} \end{cases}$$

$$\beta = \frac{\alpha}{a} \qquad a = \text{semiaxis major of satellite's orbit}$$

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1) See page 73, footnote.

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α = semiaxis major } of section of shadow-cone at distance a
$\varkappa'$ = ellipticity } from Jupiter's centre.
X = a quantity depending on perturbations in longitude
t' = observed duration.

It will be seen that $\frac{\beta}{T}$ can be considered as rigorously known. T itself, which is the maximum duration of an eclipse, is however not known.

If the immersion and emersion of the same eclipse are observed, an approximate knowledge of X is sufficient. In the most general case however immersions and emersions of different eclipses, in which X has different values, must be combined. X can then not be considered as known but contains several unknown quantities, which must be determined from the same observations. Consequently there is a considerable loss of weight in the determination of s_0.

This is however not the most serious objection to the determination of the latitudes from eclipse observations.

The chief difficulty lies in the determination of the duration itself. For the visual observations, modern as well as ancient, this determination is so mixed up with errors of all kinds, that it is very difficult, if not impossible, to derive any trustworthy results from these observations. The observed dissappearance and reappearance of the satellite do not coincide with its immersion in and emersion from the geometrical shadow-cone, but differ from these epochs for a number of reasons, of which it will be sufficient here to mention the most important ones. These are firstly the effect of the penumbra, of the refraction of the sun's rays in Jupiter's atmosphere, of the different albedo of the satellites. Further the aperture and quality of the instrument, the keenness of sight of the observer, the angular distance from the disc of Jupiter, the transparancy of the atmosphere have a large and unknown influence. All these circumstances make that, to use Laplace's words [1]), the geometrical shadow must be replaced by a fictitious shadow, which is not only different for different satellites, but also for different instruments and different observers, and even varies with the zenith-distance and the atmospheric definition. To derive a trustworthy result from the visual observations, it would be necessary to determine the form of this fictitious shadow-cone for each satellite, and for each observer and each instrument separately, and also to determine how it varies with the various circumstances of the observation. It is evident from these considerations that the visual eclipse observations are incapable of giving more than a rough approximation to the values of the inclinations and nodes of the orbits of the satellites.

With the photometric observations the conditions are much better. It is to be expected that the instant at which the light is reduced to any definite proportion of the light of the uneclipsed satellite will be determined from these observations free from most of the errors which affect the visual observations.

We must now again consider the equation (2). The right hand member contains the observed duration t' and the quantities X and T, which are supposed to be determined with sufficient accuracy from the observations.

The left hand member contains the unknown latitude, and the factor $(1 + \varkappa')$. This factor has been brought into evidence by transposing it from the denominator of the right hand member, where it generally appears, to the left hand member, with the intention to show that the observations of the duration of eclipses can with the same right be considered as determinations of this factor as of the latitude, and they must be considered as determinations of that one of these two quantities, of which our present knowledge is least accurate.

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1) *Mécanique Céleste*, tome IV, page 105.

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The ellipticity $\varkappa'$ of the section of the shadow-cone is very nearly equal to the ellipticity $\varkappa$ of Jupiter [1]), to which it stands in a known relation (see *Tisserand*, IV, page 74). The values for $\varkappa$ which have been determined by different observers have been collected by Dr. F. Cohn (*Astronomische Nachrichten* 3404, pages 333—336). The values derived by the different observers range from $\frac{1}{13.7 \pm 0.3}$ (W. Struve) to $\frac{1}{18.1 \pm 0.2}$ (Kaiser 1856) and the final means from observations by all observers are

	$\frac{1}{\varkappa}$
Heliometer	16.53 ± 0.22
Double image micrometer	16.77 ± 0.17
Filar micrometer	15.46 ± 0.31

(The mean errors given by Cohn have been transformed to probable errors.)

It is evident that the probable errors given are no measure of the true accuracy, and that the observations must be affected by very large systematic errors. We will certainly not underestimate the real precision of the determination of $\varkappa$, if we adopt ± 1 as the probable error (including systematic errors) of the denominator. The probable error of $1 + \varkappa'$ then becomes $\pm \frac{1}{250}$.

As to the accuracy with which the latitude s_0 is determined, we need here only consider the mean epoch of the Harvard photometric observations. We can thus make use of the values derived in Chapter III without taking into account the secular changes of the latitudes.

If we denote by I and N the inclination and node of a satellite's orbit referred to the plane of Jupiter's orbit, and by P and Q the quantities $I \sin N$ and $I \cos N$, we have

$$s_0 = I \sin (v - N) = Q \sin v - P \cos v.$$

Now if we derive corrections to the adopted values of I and N or of P and Q from the corrections Δp and Δq which have been found in Chapter III, it is easy to verify that the probable errors of the corrections ΔI and $I \Delta N$ (or ΔP and ΔQ) will be the same numerically as those of Δi and $i \Delta \Omega$ (or of Δp and Δq) [2]), and therefore the *relative* probable errors will be deminished in the ratio $\frac{i}{I}$. For the present purpose it is near enough to take $I = 3°$ for all the satellites. We then get for the relative probable errors $\frac{\varrho_P}{P} = \frac{\varrho_Q}{Q} = \frac{\varrho_I}{I}$ for the four satellites:

$$\pm \tfrac{1}{600} \qquad \pm \tfrac{1}{900} \qquad \pm \tfrac{1}{1600} \qquad \pm \tfrac{1}{2500}.$$

These are much smaller than the p. e. of the factor $(1 \mp \varkappa')$.

The observations of the durations of eclipses must thus not be considered as determinations of the latitudes of the satellites, but of the form of the shadow-cone, or in other words: even if we take no account of errors of observations, the eclipse observations are absolutely uncapable of giving even remotely the same accuracy as the heliometer in the determination of the latitudes, owing to the uncertainty in the value of $\varkappa$, which determines the form of the shadow-cone.

The general conclusion to be derived from the foregoing discussions is thus:

For the determination of the unknowns, which depend on longitudes and radii vectores visual eclipse observations are inferior to the heliometer, photometric eclipse observations will most probably give a greater accuracy; the heliometric and photographic observations have however the great advantage that a considerable number of them can be more easily secured. For the inclinations and nodes we must rely solely on the heliometer (or on photographs).

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1) Damoiseau simply takes $\varkappa' = \varkappa$.

2) Assuming, of course, the probable errors of Δp and Δq to be identical.

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We must now consider the problem, which was enounced at the beginning of this Chapter, *viz.* in how far the results derived in Chapter III need to be supported by other data, in order to arrive at a determination of all the elements of such accuracy that it can serve as the basis for the formation of new tables; and what are the best methods to derive the necessary supplementary data.

For the formation of tables of the motions of any heavenly bodies two things are wanted, *viz.* firstly the elements at a certain epoch and secondly their variations with the time.

The elements and the coefficients on which depend the large periodic inequalities are, or will be, determined with great accuracy for the epoch 1891.75 by the series of observations which is discussed in this work, and by the Harvard series of photometric observations, of which the mean epoch will be approximately the same.

These two series of observations support each other in a most happy way. The latitudes, which can not be determined with great accuracy from the eclipse observations, are given with a large weight by the heliometer; the longitudes, which are determined with a smaller weight by the Cape observations, will result with high precision from the Harvard series.

The determination of the secular perturbations and the smaller periodic inequalities requires a knowledge of the masses of the satellites and of the compression of Jupiter, and these quantities must again be derived partly from the coefficients of the large periodic inequalities, partly from the most considerable secular perturbations of which the values must be derived from observations. It is well known which secular perturbations were chosen by Laplace for the determination of the masses, it will however be useful here briefly to repeat the considerations on which that choice is based.

The secular variations, which must be considered, are those of the excentricities and perijovia and of the inclinations and nodes. The instantaneous excentricities and perijovia are given by the formulae (Chapter II page 21):

$$\begin{aligned} \tfrac{1}{2}h &= E\sin\Omega = \mathbf{a}_1\sin\varpi_1 + \mathbf{a}_2\sin\varpi_2 + \mathbf{a}_3\sin\varpi_3 + \mathbf{a}_4\sin\varpi_4 \\ \tfrac{1}{2}k &= E\cos\Omega = \mathbf{a}_1\cos\varpi_1 + \mathbf{a}_2\cos\varpi_2 + \mathbf{a}_3\cos\varpi_3 + \mathbf{a}_4\cos\varpi_4 \end{aligned} \tag{3}$$

Similarly for the inclinations and nodes we have

$$\begin{aligned} -p &= i\sin\Omega = \mathbf{b}_1\sin\theta_1 + \mathbf{b}_2\sin\theta_2 + \mathbf{b}_3\sin\theta_3 + \mathbf{b}_4\sin\theta_4 \\ q &= i\cos\Omega = \mathbf{b}_1\cos\theta_1 + \mathbf{b}_2\cos\theta_2 + \mathbf{b}_3\cos\theta_3 + \mathbf{b}_4\cos\theta_4. \end{aligned} \tag{4}$$

ϖ_1, ϖ_2, ϖ_3, ϖ_4 and θ_1, θ_2, θ_3, θ_4 are what Laplace calls the *périjoves*, respectively *noeuds propres* of the four satellites. These are subject to secular variations, which depend on certain functions of the masses, commonly denoted by [0], [1], [2], [3]. As a first approximation we have

$$\frac{d\varpi_1}{dt} = -\frac{d\theta_1}{dt} = [0], \quad \frac{d\varpi_2}{dt} = -\frac{d\theta_2}{dt} = [1], \quad \frac{d\varpi_3}{dt} = -\frac{d\theta_3}{dt} = [2], \quad \frac{d\varpi_4}{dt} = -\frac{d\theta_4}{dt} = [3].$$

The coefficients **a** and **b** in the equations (3) and (4) are functions of the unknown masses and elements, and must be considered as only approximately known. Adopting Souillart's numbers we have

$$\begin{aligned} h_1 &= 2E_1\sin\Omega_1 = +8.''8\sin\varpi_1 - 2.''5\sin\varpi_2 + 8.''1\sin\varpi_3 + 4.''6\sin\varpi_4 \\ h_2 &= 2E_2\sin\Omega_2 = +0.2 \quad +45.2 \quad +79.7 \quad +42.6 \\ h_3 &= 2E_3\sin\Omega_3 = 0.0 \quad -1.9 \quad +462.9 \quad +241.9 \\ h_4 &= 2E_4\sin\Omega_4 = 0.0 \quad 0.0 \quad -59.1 \quad +2995.7 \end{aligned} \tag{5}$$

and the same formulae for k_1 k_2 k_3 k_4 if the sines are changed to cosines.

In the expression for h_4 the coefficient of sin ϖ_4 is by far the most important, and consequently a very good first approximation to the value of ϖ_4 can be derived by taking $\varpi_4 = \Omega_4$.

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For the latitudes we have:

$$(6)\qquad \begin{aligned} -p_1 &= i_1 \sin \Omega_1 = +4.78 \sin \theta_1 + 33.70 \sin \theta_2 + 3.9 \sin \theta_3 - 1.4 \sin \theta_4 \\ -p_2 &= i_2 \sin \Omega_2 = -0.1 + 1689.9 + 102.3 + 15.7 \\ -p_3 &= i_3 \sin \Omega_3 = 0.0 - 57.4 + 644.2 + 97.4 \\ -p_4 &= i_4 \sin \Omega_4 = 0.0 - 1.5 + 125.3 + 811.4. \end{aligned}$$

Here we can take as a first approximation $\theta_2 = \Omega_2$. The only quantities for which good first approximations can be derived directly from the observations are thus θ_2 and ϖ_4. If these are determined for two epochs separated by a sufficiently long interval of time, we are able to determine $\frac{d\theta_2}{dt}$ and $\frac{d\varpi_4}{dt}$ or the functions [1] and [3].

The following table shews in how far each of the quantities $\square$ depends on each of the masses. Thus *e. g.* $\frac{975}{1000}$ of the motion of θ_1 or ϖ_1 is due to the compression of Jupiter, and $\frac{16}{1000}$ to the influence of satellite II, etc.

	Yearly motion.	Jb^2	$\frac{1}{M}$	m_1	m_2	m_3	m_4
[0] $= \frac{d\varpi_1}{dt} = -\frac{d\theta_1}{dt}$	51.5	0.975	0.000	—	0.016	0.008	0.001
[1] $= \frac{d\varpi_2}{dt} = -\frac{d\theta_2}{dt}$	12.0	0.823	0.001	0.040	—	0.130	0.006
[2] $= \frac{d\varpi_3}{dt} = -\frac{d\theta_3}{dt}$	2.56	0.752	0.015	0.019	0.126	—	0.088
[3] $= \frac{d\varpi_4}{dt} = -\frac{d\theta_4}{dt}$	0.736	0.363	0.119	0.007	0.031	0.480	—

This table shews that $\frac{d\theta_2}{dt}$ depends almost exclusively on the constant Jb^2 and this constant can thus be determined from the observed value of $\frac{d\theta_2}{dt}$. Once this is known the observed value of $\frac{d\varpi_3}{dt}$ will give m_3. The mass of satellite II is known from the large periodic inequality in the motion of the first satellite. The large inequality of satellite II gives $1.1\, m_3 + 0.6\, m_1$, which combined with the determination of m_3 from $\frac{d\varpi_3}{dt}$ gives m_1.

Thus all the masses are determined with the exception of m_4. This must be derived from the coefficient of $\sin \varpi_4$ in h_3 and k_3, which is approximately proportional to that mass. This coefficient together with that of $\sin \varpi_3$, which is twice the *excentricité propre* of the third satellite, must be determined from the values of E_3 at two conveniently chosen epochs.

When a good first approximation is thus obtained for the values of all the unknowns, the coefficients of the equations (3) and (4) and the quantities $\square$ are computed anew and a new approximation is made. Such is, very summarily, Laplace's method.

In the present case we must take for one of the epochs, at which the elements are determined 1891.75.

I will first consider the determination of $\frac{d\theta_2}{dt}$.

It is evident from what has been said above that, for a determination of θ_2 or Ω_2 to be compared with that of 1891.75, we cannot rely on ancient eclipse observations. It will be necessary to use heliometric or photographic observations.

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Prof. W. SCHUR has made heliometer observations of position-angles between satellite II and the planet (*Nova acta der Kais. Leop. Carol. Acad. Vol.* 45, Nr. 3). The mean epoch is 1878.9; thus the interval is 12.8 years and the motion of ϑ_2 in this interval is 154°.

Schur gives for the determination of IdN a probable error of $\pm 74''.6$. As we do not require however the node N with reference to Jupiter's orbit, but the node Ω referred to Jupiter's equator, this p. e. must be increased in the ratio $\frac{I}{i}$. We then get for the p. e. of Ω_2 in 1878.9: $\pm 2°.94$. On the other hand we have found in Chapter III for the p. e. of Ω_2 in 1891.7: $\pm 0°.37$. We will thus get the difference of 154° with a p. e. of $\pm 3°$ or about $\frac{1}{50}$.

Even if this p. e. could be considered as a measure of the true accuracy, it would not be very satisfactory. The results can moreover not be expected to be free from systematic errors. No details are published about the determination of the equatorial adjustment and the index-error. Prof. Schur's discussion of these observations shews further very clearly, that the observations of distances and position-angles between a satellite and the planet's disc are indeed (as was to be expected) affected by very large systematic errors, which Prof. Schur explains by a difference in pointing on the two edges of the planet's limb.

It thus appears necessary that a new series of observations shall be made, similar to the one here discussed. Now it is evident that, if tables are based on a value of this secular variation which is determined from two epochs with an interval of n years, these tables will only give the same accuracy in the computed latitudes as the observations on which they were based, for n years after the last epoch. It is thus, from one point of vue, advantageous to have as long an interval between the two epochs as is possible. On the other hand however the need for better tables is so urgent, especially as the existing continuations of Damoiseau's tables expire in 1910, that it is, in my opinion, not justifiable to defer the determination of the necessary elements any longer. It is also an evident advantage for the determination of secular variations, if the circumstances are as equal as possible at the two epochs. If a new series of observations was taken with the mean epoch 1903.7, the interval would be 12 years, *i. e.* a whole revolution of Jupiter, and the motion of Ω_2 would be 144°. If we suppose the new series to have a somewhat larger weight than the present one, this motion of 144° could be determined with a p. e. a little smaller than $\pm 0°.5$, or about $\frac{1}{300}$. This is also the accuracy with which the constant Jb_2 could be determined.

The probable error in the first term of $\frac{d\varpi_4}{dt}$ would then also be $\frac{1}{300}$ of its amount or ± 0.0012. It remains to be seen with what accuracy the value of $\frac{d\varpi_4}{dt}$ itself can be determined.

In this case we must have recourse to ancient eclipses. If *e. g.* 50 eclipses of satellite IV with a mean epoch about 1740 were rereduced, the interval would be about 150 years, and the motion of ϖ_4 110°. It is not probable that it will be possible to go still further back. Referring to the probable errors given on page 75, it will be seen that we can not expect to get for the quantities h and k a smaller p. e. than $\pm 0°.01$, and even this will probably be too optimistic an estimate. This would correspond to a p. e. of $\pm 0°.7$ in Ω_4 or ϖ_4. For the epoch 1891.7 we found in Chapter III a p. e. of $\pm 0°.24$. From the photometric eclipses we can probably expect such a good determination that it can be considered as absolutely correct compared with that at the epoch 1740. The p. e. of the difference of 110° will then be $\pm 0°.7$ or about $\frac{1}{150}$.

We have thus

p. e. of $\frac{d\varpi_4}{dt}$	$\pm$ 0.0067
p. e. of first term	$\pm$ 0.0012
from which p. e. of last term	$\pm$ 0.0068 or $\frac{1}{70}$

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This is thus the accuracy with which m_3 can be determined. This being known, the determination of m_1 offers no difficulties. The probable error of the value of x_2 or of $1.1\ m_3 + 0.6\ m_1$, wich can be derived from the Harvard observations, will probably be negligible as compared with the p. e. of m_3. Therefore the p. e. of the determination of $0.6\ m_1$ will be the same as that of $1.1\ m_3$, or m_1 will be determined to $\frac{1}{8}$ of its amount (adopting Souillart's masses). If we remember the present disagreement between the values of m_1 given by different investigators (see Introduction page 2), it appears that this means a real improvement of our knowledge of this mass.

It remains to discuss the determination of m_4. This is determined from the ratio $\frac{\mathbf{a}_4'''}{\mathbf{a}_3'''}$ i. e. the ratio of the coefficients of $\sin \varpi_4$ and $\sin \varpi_3$ in the expression for h_3 (see formulae 3). We have approximately

$$h_3 = \mathbf{a}_3''' \sin \varpi_3 + \mathbf{a}_4''' \sin \varpi_4 = 2E_3 \sin \Omega_3;$$
$$k_3 = \mathbf{a}_3''' \cos \varpi_3 + \mathbf{a}_4''' \cos \varpi_4 = 2E_3 \cos \Omega_3;$$

Now it is evident that the most favourable conditions for the determination of $\mathbf{a}_3'''$ and $\mathbf{a}_4'''$ would be offered by the combination of two epochs, at one of which $\varpi_3 = \varpi_4$ while at the other $\varpi_3 = \varpi_4 \pm 180°$. If at one of the epochs the difference $\varpi_3 - \varpi_4$ is a, which is supposed different from 0° and 180°, it will still be advantageous to choose the other epoch so that the difference shall be $a \pm 180°$. As the relative yearly motion of ϖ_3 and ϖ_4 is 1°.8, the two epochs should be about 100 years apart. It will thus be desirable to determine the quantities h_3 and k_3 for a mean epoch near 1790 by the rereduction of as many eclipse observations of the third satellite as are available. If we adopt for the accuracy with which E_3 can be determined at that epoch ± 0°.010 we get for the p. e. of m_4 $\frac{1}{14}$ of its amount.

A few words must be said about the new series of observations. The principal aim of this series will be the determination of $\delta\Omega_2$. Thus, if heliometer observations are taken, it will only be strictly necessary to observe position-angles of satellite II combined with the three other satellites. If this was done, the only unknowns would be p_2 and q_2. The p's and q's belonging to the other satellites, as well as the unknowns h, k, x, ε for the four satellites, would then necessarily be considered as known by their values at the epoch 1891.75 combined with approximate values of the secular perturbations. The consequence would be a considerable loss of real accuracy in the determination of p_2 and q_2, which would have to be compensated by increasing the number of observations. Moreover, after the derivation of $\delta\Omega_2$ and thence of Jb_2, most probably the adopted approximations of the secular variations would require some correction, and a new solution would be necessary. It appears thus far more practical and economical, at once to determine the values of all the unknowns p ad q independently for the new epoch, by observing position-angles of the whole set of six combinations on every night. As to the unknowns h, k, x, ε, their influence on the position-angles is so small that the approximate values will be quite sufficiently accurate. It will thus not be necessary to observe the distances with high precision. Approximate measures of distance should however always be taken, as a means to detect eventual mistakes in the identification of the satellites.

In the case of photographic observations naturally the four satellites are always observed together.

Both in the case of heliometric and photographic observations, the value of the whole series will depend on the freedom from systematic errors of the determination of the instantaneous index-error, respectively orientation of the plate. The means by which this determination is to be effected must be left to the observers. It may be remarked however that the standard stars used in 1891 will again be favourably situated, but their distance and position-angle is now affected by 12 years of unknown proper motion, and a new determination will be necessary, before they can be used for the derivation of the index-error.

If the observations are spread over three oppositions (1902, 1903, 1904), and if in the case of heliometer observations distances too are observed, the new series will also give a determination of the unknowns h, k, and x. As we have supposed the total weight of the new series to be slightly superior to that of the

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series of 1891, the probable errors of these quantities will be slightly smaller than those of the quantities F (page 69 or 75). These unknowns would thus be determined with a much greater accuracy than in the present work. It has however already been pointed out, that we may reasonably expect a very accurate determination of these unknowns from the Harvard series of photometric observations. If this expectation is fulfilled the determination for the epoch 1903.7 becomes unnecessary.

The condition that the weight of the new series shall be slightly larger than of the present series, will be fulfilled if say 40 complete sets of heliometer observations (1 set = 6 pairs) or 120 photographic plates are secured. There will be no difficulty in securing even a considerably larger number if the observations are spread over three oppositions.

Before recapitulating the principal conclusions arrived at in the foregoing discussions, I will briefly consider a method for the determination of the constant Jb^2, which at first sight appears to offer great advantages.

The node and perijove of the fifth satellite are subject to very large secular perturbations arising from the compression of Jupiter.

Dr. Cohn finds a yearly motion of the node of $911.^{\circ}7 \pm 5^{\circ}.2$ (*Astronomische Nachrichten* 3403—4.) This motion is derived from observations extending over only $2\frac{1}{2}$ years (1892 Sept.—1895 April), thus we may expect a very much smaller probable error if observations made during the present series of favourable oppositions of Jupiter are compared with those of 1892—5. Here however we meet with a new difficulty, which was also pointed out by Cohn.

The potential of a compressed planet is (*Tisserand* II page 322):

$$V = \frac{fM}{r} + \frac{fMJb^2}{r^3}\left(\tfrac{1}{3} - \mu^2\right) + \frac{fMKb^4}{r^5}\left(\tfrac{1}{10} - \mu^2 + \tfrac{7}{6}\mu^4\right) + \ldots\ldots \tag{8}$$

where

f = the Gaussian constant of attraction,
M = total mass,
$\mu = \sin\lambda$, λ = latitude referred to plane of equator,
b = equatorial diameter of free surface.

J and K are constants depending on the inner distribution of the mass, and are expressed by definite integrals. Their numerical value is independent of the particular units of length and mass which are employed [1]).

The ratio K/J is not known; it must however be of the order of magnitude of $2\varkappa$ [2]) ($\varkappa$ = ellipticity of free surface) Thus we can take approximately (taking $\varkappa = \frac{1}{15}$)

$$\frac{K}{J} = 0.13.$$

From the expression (8) of the potential we derive for the motion of the node

$$\frac{d\vartheta}{dt} = -n\left(J\frac{b^2}{a^2} + K\frac{b^4}{a^4} + \ldots\right) \tag{9}$$

For the fifth satellite, adopting Cohn's values of a and b, we find that the second term is about $\frac{1}{50}$ of the first. It can thus not be neglected in a determination of Jb^2. For the second satellite, from the motion of whose node Jb^2 is determined, the second term of the expression (9) is only $\frac{1}{600}$ of the first, and can be neglected.

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1) The observations of the satellites can only give Jb^2, which does depend on the unit of length. Now it is customary in the theory of the satellites to adopt b as unit of length, and this probably is the explanation of the fact that this dependency on the unit of length has been so generally overlooked; as was pointed out by Cohn. Cohn's $k\left(\frac{\varrho}{\varrho_0}\right)^2$ is identical to our Jb^2.

2) In the case of a homogeneous planet it would be exactly $2\varkappa$.

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The condition that the free surface of the planet shall be an equipotential surface gives [1]).

$$\frac{\varkappa}{1-\varkappa} - \frac{\varphi}{2} = J\left(\frac{1}{3} + \frac{2}{3}\lambda^3\right) + K\left(\frac{1}{10} - \frac{4}{15}\lambda^5\right) + \ldots . \tag{10}$$

which comes in the place of the well known approximate relation

$$\varkappa - \frac{\varphi}{2} = J. \tag{11}$$

In this formula $\lambda = \frac{1}{1-\varkappa}$, and φ is the ratio between the centrifugal force and the force of gravity at the equator, which is given by the equation.

$$\varphi = \frac{\omega^2 b^3}{f M} = f\omega^2 b^3 \mathscr{M}(1 + \Sigma m) \tag{12}$$

where ω = the velocity of axial rotation expressed in parts of the radius and the mean solar day as units, and where Σm denotes the sum of the masses of the satellites.

If $\varkappa$, φ (or ω) and b were known with sufficient accuracy, the equation (10) would give a relation between J and K, by means of which K could be eliminated from the equation (9) derived from the fifth satellite. Such is however not the case, and actually the inverse course must be followed. Jb^2 being known from the motion of the second satellite, (9) determines Kb^4. Then (10) gives a relation between $\varkappa$, φ and b, in which the value (12) of φ can be substituted. $\varkappa$ can be determined from observations of the durations of eclipses with the aid of the latitudes given by the heliometer observations, as has been explained above. The velocity ω can be assumed to be known with sufficient accuracy, thus we will have an equation in which the diameter b of the free surface of the planet is the only unknown quantity [2]). I will call this the theoretical determination of the diameter.

This theoretical determination will most probably be far more reliable than the direct measurement. The latter is not only subject to large personal and instrumental errors, but the value of the diameter is also affected by the refraction of the sun's rays in Jupiter's atmosphere. Moreover there can be little doubt but the visible diameter is not that of the planet but of the layer of heavy clouds, about whose altitude over the planet's surface we know nothing.

The diameter of the planet might further be determined from the observed maximum durations of eclipses. This determination however is also affected to a large and unknown extent by the absorption and refraction of the sun's rays in Jupiter's atmosphere, and will certainly not be more accurate than the direct measurement.

The knowledge of K, besides rendering the theoretical determination of the diameter more exact, is important in itself and will probably allow some interesting conclusions to be drawn about the inner distribution of the mass of Jupiter.

We see thus that the observations of the fifth satellite can not contribute to a better knowledge of

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1) c.f. Bessel, *Theorie des Saturn-Systems.* Astronomische Nachrichten Nr. 672, page 386.

2) Laplace (*Mécanique Céleste* IV, page 126) derives φ in the equation $\varkappa - \frac{\varphi}{2} = J$ from the observed value of ω, and then solves $\varkappa$ from this equation, saying that this determination of $\varkappa$ is more accurate than that by direct measurement. Here again, in consequence of the fact that b is taken as unit, it is overlooked that the known quantity is not J but Jb^2, and that φ is not proportional to ω^2 but to $\omega^2 b^3$: $\varkappa$ is thus not the only unknown quantity in the equation (11), but the values of φ and J in this equation contain implicitly the unknown diameter b.

The reasoning given above applies of course as well when $\varkappa$ is neglected. The equation (9) for satellite V is then not required, and (10) can be replaced by (11).

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the motions of the outer satellites [1]). They are however very interesting in themselves, and will give important data relating to the constitution of the planet.

Finally I will briefly recapitulate the principal conclusions which were arrived at in this Chapter. They were the following.

It is desirable that at least 50 observations of eclipses of satellite IV (an equal number of immersions and emersions) with a mean epoch not much later than 1750, be rereduced, in order to determine the secular motion of ϖ_4.

It is desirable that a like number of observations of satellite III with a mean epoch about 1790 be discussed, to determine the term in the equation of the centre of satellite III, which depends on the mass of satellite IV.

It is desirable that a series of observations, either heliometric or photographic, be taken about the epoch 1903.7, chiefly for the determination of the motion of the node of satellite II.

The weight of this series should be slightly superior to that of the series of 1891.75 which was discussed in this work.

If heliometer observations are taken, the position-angles should be observed with the highest precision; the distances only approximately for purposes of identification.

All six combinations of the satellites should be observed.

Both for heliometric and photographic observations the determination of the index-error, respectively orientation, free from systematic errors is of the most vital importance.

I give here the quantities, very roughly computed, which determine the conditions of observation in the next three oppositions of Jupiter, to which I add for the sake of comparison the same quantities for the epoch 1891.75, (not for date of opposition, which is 1891.67).

Date of opposition	R.A.	Decl.	log Δ	L	c	$L+c$
(1891.75)	23^h	$-9°$	0.610	210°	12°	222°
1902.6	21	—18	0.608	177	65	242
1903.7	23	— 6	0.598	214	2	216
1904.8	2	+ 8	0.597	249	299	188

[1]) Perturbations of V by I, and a fortiori by the other three satellites, are insensible owing to the proximity of Jupiter and the absence of a proportionality of the mean motions to whole numbers like that which makes the perturbations of I, II and III so large.

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APPENDIX.

ON THE SOLUTION OF NORMAL EQUATIONS.

If a system of m linear equations with m unknowns be given of the form

N
$$\begin{aligned}
&[aa]\,x+[ab]\,y+[ac]\,z+[ad]\,u+\ .\ .\ .\ .=X\\
&[ba]\,x+[bb]\,y+[bc]\,z+[bd]\,u+\ .\ .\ .\ .=Y\\
&[ca]\,x+[cb]\,y+[cc]\,z+[cd]\,u+\ .\ .\ .\ .=Z\\
&[da]\,x+[db]\,y+[dc]\,z+[dd]\,u+\ .\ .\ .\ .=U\\
&.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .
\end{aligned}$$

where the quantities $[kl]$ denote any numbers, such that $[kl] = [lk]$, then the solution of these equations will again be of the same form, thus:

S
$$\begin{aligned}
&[\varkappa\varkappa]\,X+[\varkappa\beta]\,Y+[\varkappa\gamma]\,Z+[\varkappa\delta]\,U+\ .\ .\ .\ .=x\\
&[\beta\varkappa]\,X+[\beta\beta]\,Y+[\beta\gamma]\,Z+[\beta\delta]\,U+\ .\ .\ .\ .=y\\
&[\gamma\varkappa]\,X+[\gamma\beta]\,Y+[\gamma\gamma]\,Z+[\gamma\delta]\,U+\ .\ .\ .\ .=z\\
&[\delta\varkappa]\,X+[\delta\beta]\,Y+[\delta\gamma]\,Z+[\delta\delta]\,U+\ .\ .\ .\ .=u\\
&.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .
\end{aligned}$$

where $[\varkappa\lambda] = [\lambda\varkappa]$.

The method of deriving the set of equations S from the set N, which was used for the solution of the normal equations occurring in the present work, is essentially the same as that which is called *Zweites Auflösungsverfahren* by HELMERT (*Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate*). The practical value of this method appears not to have found the general appreciation, which it does, in my opinion, deserve. Also it only came to my knowledge after I had developed it independently (starting from GAUSS's method as given by OPPOLZER, *Lehrbuch zur Bahnbestimmung von Kometen und Planeten*, Vol. II), and when the manuscript of this dissertation was already in the printer's hands. That I have decided, in these circumstances, to publish this appendix in its original form, is chiefly because I hoped the more detailed explanation of the arrangement of the computations, which is here given, might possibly contribute a little towards assuring a more general adoption of the method.

The first step is the formation of the *equations of elimination. (Eliminationsgleichungen, Reducirte Normalgleichungen, Intermediate normals.)* If we solve x from the first equation N, and if we put

$$-\frac{[ab]}{[aa]} = a_b \qquad -\frac{[ac]}{[aa]} = a_c \qquad -\frac{[ad]}{[aa]} = a_d \ .\ .\ .\ .$$

we get

$$x = \frac{1}{[aa]}X + a_b\,y + a_c\,z + a_d\,u + \ .\ .\ .\ .$$

Substituting this value in the other $m-1$ equations N, and introducing the well known notations:

$$\begin{aligned}
&[bb]+a_b\,[ba]=[bb1] \qquad \text{or} \qquad [bb]+a_b\,[ab]=[bb1]\\
&[bc]+a_c\,[ba]=[bc1] \qquad\qquad\ \ [bc]+a_b\,[ac]=[bc1]\\
&[bd]+a_d\,[ba]=[bd1] \qquad\qquad\ \ [bd]+a_b\,[ad]=[bd1]
\end{aligned}$$

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and generally

$$[kl] + a_k\,[al] = [kl1]$$

we get $m - 1$ equations with $m - 1$ unknowns:

$$\begin{aligned}
&[bb1]\,y + [bc1]\,z + [bd1]\,u + \ldots = Y + a_b\,X\\
&[cb1]\,y + [cc1]\,z + [cd1]\,u + \ldots = Z + a_c\,X\\
&[db1]\,y + [dc1]\,z + [dd1]\,u + \ldots = U + a_d\,X\\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

These are again of the normal form, and can be treated in the same way as the equations N. If we put

$$-\frac{[bc1]}{[bb1]} = b'_c \qquad -\frac{[bd1]}{[bb1]} = b'_d \ \ldots\ldots$$

$$[kl2] = [kl1] + b'_k\,[bl1]$$

$$a_c + b'_c\,a_b = a'_c \qquad a_d + b'_d\,a_b = a'_d \ \ldots\ldots$$

we will get $m - 2$ equations:

$$\begin{aligned}
&[cc2]\,z + [cd2]\,u + \ldots = Z + b'_c\,Y + a'_c\,X\\
&[dc2]\,z + [dd2]\,u + \ldots = U + b'_d\,Y + a'_d\,X\\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

These are again treated in the same manner, introducing the notations:

$$-\frac{[cd2]}{[cc2]} = c''_d \ \ldots\ldots$$

$$[kl3] = [kl2] + c''_k\,[cl2]$$

$$a'_d + c''_d\,a'_c = a''_d \ \ldots\ldots$$

$$b'_d + c''_d\,b'_c = b''_d \ \ldots\ldots$$

We then get $m - 3$ equations:

$$\begin{aligned}
&[dd3]\,u + \ldots = U + c''_d\,Z + b''_d\,Y + a''_d\,X\\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

which are again reduced to $m - 4$, and these to $m - 5$, and so on till there is only one equation with one unknown left. For the present purpose it will be sufficient to suppose the number of unknowns to be four.

The equations of elimination are then:

E

$$\begin{aligned}
[aa]\,x + [ab]\,y + [ac]\,z + [ad]\,u &= X\\
[bb1]\,y + [bc1]\,z + [bd1]\,u &= Y + a_b\,X\\
[cc2]\,z + [cd2]\,u &= Z + b'_c\,Y + a'_c\,X\\
[dd3]\,z &= U + c''_d\,Z + b''_d\,Y + a''_d\,X
\end{aligned}$$

Recapitulating, we have, for the formation of the equations E, to compute the following quantities:

Ia $\quad -\frac{[ab]}{[aa]} = a_b \qquad -\frac{[ac]}{[aa]} = a_c \qquad -\frac{[ad]}{[aa]} = a_d \qquad +\frac{[as]}{[aa]} = a_s$

IIa $\quad [bb] + a_b\,[ab] = [bb1] \qquad [bc] + a_b\,[ac] = [bc1] \qquad [bd] + a_b\,[ad] = [bd1] \qquad [bs] + a_b\,[as] = [bs1]$

IIIa $\quad -\frac{[bc1]}{[bb1]} = b'_c \qquad -\frac{[bd1]}{[bb1]} = b'_d \qquad +\frac{[bs1]}{[bb1]} = b'_s$

IVa $\quad [cc] + a_c\,[ac] + b'_c\,[bc1] = [cc2] \qquad [cd] + a_c\,[ad] + b'_c\,[bd1] = [cd2] \qquad [cs] + a_c\,[as] + b'_c\,[bs1] = [cs2]$

Va $\quad -\frac{[cd2]}{[cc2]} = c''_d \qquad +\frac{[cs2]}{[cc2]} = c''_s$

VIa $\quad [dd] + a_d\,[ad] + b'_d\,[bd1] + c''_d\,[cd2] = [dd3] \qquad [ds] + a_d\,[as] + b'_d\,[bs1] + c''_d\,[cs2] = [ds3]$

VIIa $\quad a_b = a_b \qquad 1 + a_b = s_b$

VIIIa $\quad b'_c = b'_c \qquad a_c + b'_c\,a_b = a'_c \qquad 1 + a_c + b'_c\,s_b = s'_c$

IXa $\quad c''_d = c''_d \qquad b'_d + c''_d\,b'_c = b''_d \qquad a_d + b'_d\,a_b + c''_d\,a'_c = a''_d \qquad 1 + a_d + b'_d\,s_b + c''_d\,s'_c = s''_d$

I have also included the quantities which are necessary for the formation of the check-equations.

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We put, as is usual:

$$[aa]+[ab]+[ac]+[ad]=[as]$$
$$[ba]+[bb]+[bc]+[bd]=[bs]$$
$$[ca]+[cb]+[cc]+[cd]=[cs]$$
$$[da]+[db]+[dc]+[dd]=[ds]$$

We then have the following check-equations, all of which are very easily verified, and may be given here without proof:

Ib $\quad a_b+a_c+a_d+a_s=+1$

IIb $\quad [bb1]+[bc1]+[bd1]=[bs1]$

IIIb $\quad b'_c+b'_d+b'_s=+1$

IVb $\quad [cc2]+[cd2]=[cs2]$

Vb $\quad c''_d+c''_s=+1$

VIb $\quad [dd3]=[ds3]$

VIIb $\quad 1+a_b=s_b$

VIIIb $\quad 1+b'_c+a'_c=s'_c$

IXb $\quad 1+c''_d+b''_d+a''_d=s''_d$

There is another very important check on the second members of the equations E. If they are multiplied respectively by a_s, b'_s, c''_s and 1, and then added, the coefficients of U, Z, Y and X in the sum will each be equal to unity. In other words we have the following relations:

X
$$+1=+1$$
$$c''_s+c''_d=+1$$
$$b'_s+c''_s\,b'_c+b''_d=+1$$
$$a_s+b_s\,a_b+c''_s\,a'_c+a''_d=+1$$

which can easily be proved with the aid of the check-equations Ib, IIIb and Vb and the definitions VIIIa and IXa of the quantities a'_c, b''_d and a''_d.

If we add the four equations X we get

XI $\quad a_s+b'_s\,s_b+c''_s\,s'_c+s''_d=4=$ *number of unknowns.*

It will be sufficient in practice to apply the test XI, and only to use X to localise the error, in the cases when XI is not satisfied.

We now come to the second step, *viz.* the derivation of the coefficients of the solution S from the equations of elimination E.

To begin with we write these equations thus:

E′
$$x-a_b\,y-a_c\,z-a_d\,u=\frac{1}{[aa]}\,X$$
$$y-b'_c\,z-b'_d\,u=\frac{1}{[bb1]}\,Y+\frac{a_b}{[bb1]}\,X$$
$$z-c''_d\,u=\frac{1}{[cc2]}\,Z+\frac{b'_c}{[cc2]}\,Y+\frac{a'_c}{[cc2]}\,X$$
$$u=\frac{1}{[dd3]}\,U+\frac{c''_d}{[dd3]}\,Z+\frac{b''_d}{[dd3]}\,Y+\frac{a''_d}{[dd3]}\,X$$

The last equation E′ gives at once the last equation S, we have thus:

$$[\delta\delta]=\frac{1}{[dd3]}\qquad[\delta\gamma]=\frac{c''_d}{[dd3]}\qquad[\delta\beta]=\frac{b''_d}{[dd3]}\qquad[\delta\alpha]=\frac{a''_d}{[dd3]}$$

and

(1) $$u=[\delta\delta]\,U+[\delta\gamma]\,Z+[\delta\beta]\,Y+[\delta\alpha]\,X.$$

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Multiplying this value of u by c''_d and adding it to the last equation but one of E′, we get:

$$z = c'_d\,[\delta\delta]\;U + \left\{\frac{1}{[cc2]} + c''_d\,[\delta\gamma]\right\} Z + \left\{\frac{b'_c}{[cc2]} + c''_d\,[\delta\beta]\right\} Y + \left\{\frac{a'_c}{[cc2]} + c''_d\,[\delta x]\right\} X$$

or

$$z = [\gamma\delta]\;U + [\gamma\gamma]\;Z + [\gamma\beta]\;Y + [\gamma x]\;X, \tag{2}$$

which is the last equation but one of S.

It is easily verified that $[\gamma\delta] = [\delta\gamma]$, as it should be.

Again adding $b'_d\,u + b'_c\,z$ [where for u and z their values (1) and (2) must be taken] to the second equation E′, we get:

$$y = \ldots\ldots + \left\{\frac{1}{[bb1]} + b'_d\,[\delta\beta] + b'_c\,[\gamma\beta]\right\} Y + \left\{\frac{a_b}{[bb1]} + b'_d\,[\delta x] + b'_c\,[\gamma x]\right\} X.$$

It is not necessary to compute the coefficients of U and Z, as we know that they will be $[\beta\delta] = [\delta\beta]$ and $[\beta\gamma] = [\gamma\beta]$, which have been computed before. We have thus:

$$y = [\beta\delta]\;U + [\beta\gamma]\;Z + [\beta\beta]\;Y + [\beta x]\;X. \tag{3}$$

Lastly adding to the first equation E′ the sum $a_d\,u + a_c\,z + a_b\,y$ [where for u, z and y we take their values (1), (2) and (3)], we get:

$$x = \ldots\ldots + \left\{\frac{1}{[aa]} + a_d\,[\delta x] + a_c\,[\gamma x] + a_b\,[\beta x]\right\} X,$$

where it is unnecessary to compute other coefficients than that of X.

Recapitulating we have, for the solution of the equations E, to compute the following quantities, among which I include those necessary for the formation of the check-equations:

XIIa

$$\begin{array}{lllll} \frac{1}{[dd3]} = [\delta\delta] & \frac{c''_d}{[dd3]} = [\delta\gamma] & \frac{b''_d}{[dd3]} = [\delta\beta] & \frac{a''_d}{[dd3]} = [\delta x] & \frac{s''_d}{[dd3]} = [\delta\sigma] \\ & \frac{1}{[cc2]} & \frac{b'_c}{[cc2]} & \frac{a'_c}{[cc2]} & \frac{s'_c}{[cc2]} \\ & & \frac{1}{[bb1]} & \frac{a_b}{[bb1]} & \frac{s_b}{[bb1]} \\ & & & \frac{1}{[aa]} & \frac{1}{[aa]} \end{array}$$

XIIIa $\quad \frac{1}{[cc2]} + c''_d\,[\delta\gamma] = [\gamma\gamma] \qquad \frac{b'_c}{[cc2]} + c'_d\,[\delta\beta] = [\gamma\beta] \qquad \frac{a'_c}{[cc2]} + c'_d\,[\delta x] = [\gamma x] \qquad \frac{s'_c}{[cc2]} + c''_d\,[\delta\sigma] = [\gamma\sigma]$

XIVa $\quad \frac{1}{[bb1]} + b'_d\,[\delta\beta] + b'_c\,[\gamma\beta] = [\beta\beta] \qquad \frac{a_b}{[bb1]} + b'_d\,[\delta x] + b'_c\,[\gamma x] = [\beta x] \qquad \frac{s_b}{[bb1]} + b'_d\,[\delta\sigma] + b'_c\,[\gamma\sigma] = [\beta\sigma]$

XVa $\quad \frac{1}{[aa]} + a_d\,[\delta x] + a_c\,[\gamma x] + a_b\,[\beta x] = [xx] \qquad \frac{1}{[aa]} + a_d\,[\delta\sigma] + a_c\,[\gamma\sigma] + a_b\,[\beta\sigma] = [x\sigma]$

and we have the following check-equations:

XIIb

$$\begin{array}{r} [\delta\delta] + [\delta\gamma] + [\delta\beta] + [\delta x] = [\delta\sigma] \\ \frac{1}{[cc2]} + \frac{b'_c}{[cc2]} + \frac{a'_c}{[cc2]} = \frac{s'_c}{[cc2]} \\ \frac{1}{[bb1]} + \frac{a_b}{[bb1]} = \frac{s_b}{[bb1]} \end{array}$$

XIIIb $\quad [\gamma\delta] + [\gamma\gamma] + [\gamma\beta] + [\gamma x] = [\gamma\sigma]$

XIVb $\quad [\beta\delta] + [\beta\gamma] + [\beta\beta] + [\beta x] = [\beta\sigma]$

XVb $\quad [x\delta] + [x\gamma] + [x\beta] + [xx] = [x\sigma]$

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The first three of these are self-evident. To prove XIII*b*, remark that it can be written:

$$\left.\begin{array}{l} c''_d\,[\delta\delta] + c''_d\,[\delta\gamma] + c''_d\,[\delta\beta] + c''_d\,[\delta\alpha] \\ + \dfrac{1}{[cc2]} + \dfrac{b'_c}{[cc2]} + \dfrac{a'_c}{[cc2]} \end{array}\right\} = \left\{\begin{array}{l} c''_d\,[\delta\sigma] \\ + \dfrac{s'_c}{[cc2]} \end{array}\right.$$

which is a combination of the first and second equation XII*b*. The two remaining equations XIV*b* and XV*b* are proved by a similar reasoning.

If the values S of the unknowns are substituted in the equations N, and then these equations are added, we find by comparing the coefficients of X, Y, Z, U in the left and right hand members of the sum:

$$\begin{array}{l} [as]\,.\,[\alpha\alpha] + [bs]\,.\,[\alpha\beta] + [cs]\,.\,[\alpha\gamma] + [ds]\,.\,[\alpha\delta] = +1 \\ [as]\,.\,[\beta\alpha] + [bs]\,.\,[\beta\beta] + [cs]\,.\,[\beta\gamma] + [ds]\,.\,[\beta\delta] = +1 \\ [as]\,.\,[\gamma\alpha] + [bs]\,.\,[\gamma\beta] + [cs]\,.\,[\gamma\gamma] + [ds]\,.\,[\gamma\delta] = +1 \\ [as]\,.\,[\delta\alpha] + [bs]\,.\,[\delta\beta] + [cs]\,.\,[\delta\gamma] + [ds]\,.\,[\delta\delta] = +1 \end{array}$$

Adding these four equations, we get as a final control:

XVI[1]) $$[s\sigma] = [as]\,[\alpha\sigma] + [bs]\,[\beta\sigma] + [cs]\,[\gamma\sigma] + [ds]\,[\delta\sigma] = 4 = \textit{number of unknows.}$$

I now proceed to explain a symmetrical arrangement of the computations, which I have found very convenient in practice.

The first step is the formation of the equations of elimination (E) and the „*multiplicators*" a_b, a_c, . . b'_c . . etc. The computation is arranged in two sets of $m+1$ vertical columns and $\dfrac{m\,[m+3]}{2} - 1$ horizontal spaces one set for the left hand and one for the right hand members of the equations, the last column of each set being reserved for the checks. The first horizontal space is included between two horizontal lines, then two spaces are left blank, then one included between two lines, then three left blank etc. The arrangement will be seen from the example which is appended. The normal equations are written in the horizontal spaces marked with N_1 N_2 N_3 N_4 in the example, *viz.* the first normal in the top space, the second normal, without the first term, immediately below it, the third normal, without its first two terms, in the space below the second double horizontal line, . . etc. In the right hand set of columns are written the coefficients of U, Z, Y, X in the normals, and their sums, i. e. $+1$ in each case.

I. We now proceed to compute the multiplicators a_b, a_c, a_d. These are written in the column under $[aa]$, opposite the blank spaces immediately below the normal equations. a_s is written on the bottom line of the form. The check:

$$a_b + a_c + a_d + a_s = 1$$

is applied as soon as these quantities are computed.

II. We now take in Crelle's tables the page of the multiplicator a_b and write its products by each of the terms of the first normal equation (excepting the first term) in the blank space opposite a_b, *i.e.* immediately below the corresponding terms of the second normal. Adding the numbers which are in the same vertical column we get the second equation E, which finds its place in the space between the two horizontal lines, in the example marked with E_2. The check:

$$[bb1] + [bc1] + [bd1] = [bs1]$$

is then applied.

VII. The same operation is carried through the right hand members, which here consists in simply

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1) This equation is also given by Prof. A. S. Flint, *Annals of Mathematics*, Vol. IV, N⁰. 6.

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writing a_b in the columns headed X and σ. For the right hand member of equation E_2 we get by addition the coefficients 1 and a_b, and the check-quantity $s_b = 1 + a_b$.

III. Next compute the multiplicators b_c, b'_d and the check-quantity b'_s, which are written vertically under $[bb1]$ opposite the second blank spaces below the third and fourth normals. Apply the check:

$$b'_c + b'_d + b'_s = +1$$

IV and VIII. Then take in Crelle the page of a_c and write its products by the terms of E_1 (excepting the first two) under the corresponding terms of N_3 in both left and right hand members. Next, taking the page of b'_c in Crelle, write its products by the terms of E_2 in the next line, and add. This gives the equation E_3. Apply the checks:

$$[cc2] + [cd2] = [cs2]$$
$$1 + b'_c + a'_c = s'_c$$

V. Form the quantities c''_d and c''_s, which are written in the column of $[cc2]$, and apply the check:

$$c''_d + c''_s = +1$$

VI and IX. Form, by the aid of Crelle, the products of a_d by the last term and the check-quantity of E_1, similarly of b'_d by the terms of E_2 and of c''_d by the terms of E_3. Addition gives the equation E_4. Apply the checks:

$$[dd3] = [ds3]$$
$$1 + c''_d + b''_d + a''_d = s''_d$$

If it is desired to apply the check-equations X, this can be done in a separate computation, as is shewn in the example. If we wish only to use XI, which will generally be the case in practice, the products of 1, s_b, s'_c, and s''_d by a_s, b'_s, c''_s and 1 can be conveniently written in the margin, which is also done in the example.

We now come to the second step, *viz.* the determination of the solution S from the equations E. For this part of the computation a form is arranged, in every respect similar to that for the left hand members of the equations E.

In the first place the equations E must be transformed to the equations E', which are written in the same places, which were occupied by the normal equations in the first part of the computation. Thus the five quantities

$$1 \qquad c''_d \qquad b''_d \qquad a''_d \qquad s''_d$$

are multiplied by $\frac{1}{[dd3]}$ and the products are written in the space between the first pair of horizontal lines. The check:

$$[\delta\delta] + [\delta\gamma] + [\delta\beta] + [\delta\alpha] = [\delta\sigma]$$

is applied.

The four coefficients of the right-hand member of E_3 are similarly multiplied by $\frac{1}{[cc2]}$ and the products written on the line, which in the example is marked with E'_3. We have the check:

$$\frac{1}{[cc2]} + \frac{b'_c}{[cc2]} + \frac{a_c}{[cc2]} = \frac{s'_c}{[cc2]}$$

The coefficients of E_2 are multiplied by $\frac{1}{[bb1]}$ and the products written in the space E'_2. The check is:

$$\frac{1}{[bb1]} + \frac{a_b}{[bb1]} + \frac{s_b}{[bb1]}$$

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Lastly in the space marked with E'_1 we write the coefficient of X in the equation E'_1 *viz:* $\frac{1}{[aa]}$ and the same quantity is placed in the check-column.

The mechanism of the computation is precisely similar to that of the first part; with the exception that the multiplicators are taken from that first part. They are written in the same places which they occupied there, but in a different order, so that the quantities, which were in the first part in a line inclined towards the right, now come in a vertical column, and vice versa, while the quantity which was in the left hand top corner now comes in the right hand bottom corner.

XIII. The first multiplicator thus being c'_d its products with the terms of S_4, which is the same as E'_4, are written below the corresponding terms of E'_3 and added to them. This gives the terms of S_3. Now the check

$$[\gamma\delta] + [\gamma\gamma] + [\gamma\beta] + [\gamma\alpha] = [\gamma\sigma]$$

must be applied.

The last three of the four quantities in the left hand member of this equation are in the same horizontal line with $[\gamma\sigma]$, but $[\delta\gamma]$ must be taken from the horizontal line of S_4, in the column of $[\gamma\gamma]$.

XIV. The equation S_2 is formed by adding to E'_2 the products of b'_d by the terms of S_4 and of b'_c by those of S_3. The check equation is:

$$[\beta\delta] + [\beta\gamma] + [\beta\beta] + [\beta\alpha] = [\beta\sigma],$$

where $[\beta\alpha]$ and $[\beta\beta]$ are taken from the horizontal space S_2, then going upwards along the vertical column of $[\beta\beta]$ we get $[\gamma\beta]$ from the equation S_3 and $[\beta\delta]$ from equation S_4.

XV. The formation of the last term $[\alpha\alpha]$ of equation S_1 is exactly similar to the formation of $[dd3]$ in the first part of the computation, the quantities required for the check-equation:

$$[\alpha\delta] + [\alpha\gamma] + [\alpha\beta] + [\alpha\alpha] = [\alpha\sigma]$$

are all in the same vertical column as $[\alpha\alpha]$, respectively in the horizontal spaces S_4, S_3, S_2, S_1.

XVI. For the final check $[s\sigma] = 4$ the products $[\delta\sigma].[ds]$, $[\gamma\sigma].[cs]$, $[\beta\sigma].[bs]$, $[\alpha\sigma].[as]$ can be written in the margin, as is shewn in the example.

For the multiplicators a_b, a_c . . b'_c . . it will be always advisable to take *three* decimal places, and no more. I have sometimes been in the habit of taking into account halves, or even quarters of the last decimal, by a small interpolation which is done at sight while using Crelle's tables. By this device it is possible through a very long series of computations with Crelle always to keep three significant figures correct. The number of decimal places to be taken in the equations E' and in the solution must depend on the seize of the coefficients of the normal equations. Four significant figures in the quantities $\frac{1}{[aa]}$, $\frac{1}{[bb1]}$ . . , will generally be most convenient, and in the final solution the last figure can be dropped without any real loss of accuracy. The uncertainties of the resulting values of the unknowns will then very seldom be more than one or two units of the third significant figure.

In many cases a greater accuracy is wanted. Recourse can be had to the use of logarithms or a calculating machine, but this seems not advisable, as an almost unlimited degree of refinement can very easily be obtained by the same method with Crelle. If the values of the unknowns are substituted in the normal equations N, these will not be rigorously satisfied, but the absolute terms resulting from the substitution will be $X - \delta X$, $Y - \delta Y$ . . . If then δX, δY . . , are substituted in the solution S, we will get corrections δx, δy to the first values of x, y . . . The relative uncertainty in the corrections δx, δy . . . will be the same as in the original values x, y . . . , but the uncertainty in the final values $x + \delta x$, $y + \delta y$ . . . is equal to the *absolute* uncertainty in δx, δy . . . , *i. e.* at least two more significant figures will be correct, than in the first values of x,

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y.... This process of successive approximation, which can of course, if necessary, be continued, requires very little additional labour, certainly incomparably less than the excess of the labour involved in a logarithmic computation over that necessary for a solution with Crelle. And in addition to this saving of labour, the liability to mistakes is also smaller than with the use of logarithms. In order to show the accuracy which can be attained by this method, working with Crelle, I will here give the different checks, employed in the solution of the system of 18 normal equations which was treated in chapter III, and there denoted by **S**. The sums of the multiplicators, such as

$$a_b + a_c + a_d \ldots\ldots\ldots + a_s = 1$$

gave the 17 results:

1.001 1.001 0.999 1.001 0.999 0.997 1.000 1.001 1.000 1.000 0.999 0.997 1.000 1.000 0.999 1.000 1.000.

Te check-equations X gave the following 18 sums:

1.000 1.000 1.000 0.999 1.000 0.999 1.001 1.000 1.001 1.002 1.000 0.999 0.998 1.001 1.001 1.001 1.001 1.000.

The equation XI gave 18.003.

The final check gave $[s\sigma] = 17.96$.

The whole of this solution took me, single handed, about fifty hours, of which about 15 were spent in searching for and correcting a mistake, which had arisen through changing the sign of one of the multiplicators in the right hand member of one of the equations E.

At that time I had not yet found the check-equations X and XI, and the error was only discovered by means of the check on the corresponding equation S. The equation XI would have shown it at a much earlier stage of the computations.

I do not think it necessary to dwell upon the general advantages of a solution in terms of the right hand members of the normal equations. A practical example of its usefulness is furnished by the present investigation.

It needs hardly be remembered that the weights of the unknowns are given by the same solution, *viz:*

$$p_x = [\alpha\alpha] \qquad p_y = [\beta\beta] \ldots\ldots \text{etc.}$$

The first part of the method here advocated, *viz.* the formation of the left hand members of the equations E, is the same as in Oppolzer's method, only with the abbreviations wich are the natural outcome of the substitution of Crelle's tables for the use of logarithms. The determination of the unknowns *and their weights* according to Oppolzer's method is certainly not shorter than it is in my method, while the fact that this second part of the computation is arranged in a precisely similar form to the first part, seems to me to be a great advantage.

In conclusion I will mention a relation, which can be often very useful as a check on the whole of the computations. If the normal equations N are formed from the equations of condition:

$$a_1 x + b_1 y + c_1 z + d_1 u = n_1$$
$$a_2 x + b_2 y + c_2 z + d_2 u = n_2$$
$$\ldots\ldots\ldots\ldots\ldots$$

and if we form the residuals by substitution of the final values of the unknowns:

$$v_1 = n_1 - \{a_1 x + b_1 y + c_1 z + d_1 u\}$$
$$v_2 = n_2 - \{a_2 x + b_2 y + c_2 z + d_2 u\}$$
$$\ldots\ldots\ldots\ldots\ldots$$

then we have the following formula (which is identical to the first of the set (12) Oppolzer II page 338):

$$[vv] = [nn] - \{Xx + Yy + Zz + Uu\}.$$

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EXAMPLE SHEWING ARRANGEMENT OF COMPUTATIONS.

(The Normal equations are marked with N_1, N_2, N_3, N_4; the equations of elimination with E_1, E_2, E_3, E_4; their transformations with E'_1, E'_2, E'_3, E'_4; the solution with S_1, S_2, S_3, S_4. An asterisk (*) refers to a check-equation.)

Normal Equations.

$$\begin{aligned} [aa]x + [ab]y + [ac]z + [ad]u &= X \\ [bb] + [bc] + [bd] &= Y \\ + [cc] + [cd] &= Z \\ + [dd] &= U \end{aligned}$$

Left hand members / *Right hand members*

	x	y	z	u	s	U	Z	Y	X	σ	
$N_1 = E_1$	$[aa]$	$[ab]$	$[ac]$	$[ad]$	$[as]$				$+1$	$+1$	a_s
N_2		$[bb]$	$[bc]$	$[bd]$	$[bs]$			$+1$		$+1$	
$-\frac{[ab]}{[aa]} = a_b$		$a_b[ab]$	$a_b[ac]$	$a_b[ad]$	$a_b[as]$				a_b	a_b	
E_2		$[bb1]$	$[bc1]$	$[bd1]$	$[bs1]$			$+1$	a_b	s_b	$b_s s_b$
N_3			$[cc]$	$[cd]$	$[cs]$		$+1$			$+1$	
$-\frac{[ac]}{[aa]} = a_c$			$a_c[ac]$	$a_c[ad]$	$a_c[as]$				a_c	a_c	
$-\frac{[bc1]}{[bb1]} = b'_c$			$b'_c[bc1]$	$b'_c[bd1]$	$b'_c[bs1]$			b'_c	$b'_c a_b$	$b'_c s_b$	
E_3			$[cc2]$	$[cd2]$	$[cs2]$		$+1$	b'_c	a'_c	s'_c	$c''_s s'_c$
N_4				$[dd]$	$[ds]$	$+1$				$+1$	
$-\frac{[ad]}{[aa]} = a_d$				$a_d[ad]$	$a_d[as]$				a_d	a_d	
$-\frac{[bd1]}{[bb1]} = b'_d$				$b'_d[bd1]$	$b'_d[bs1]$			b'_d	$b'_d a_b$	$b'_d s_b$	
$-\frac{[cd2]}{[cc2]} = c''_d$				$c''_d[cd2]$	$b''_d[cs2]$		c''_d	$c''_d b'_c$	$c''_d a'_c$	$c''_d s'_c$	
E_4				$[dd3]$	$[ds3]$	$+1$	c''_d	b''_d	a''_d	s''_d	s''_d
$+\frac{[as]}{[aa]} = a_s$		$+\frac{[bs1]}{[bb1]} = b'_s$	$+\frac{[cs2]}{[cc2]} = c''_s$								
Sum $= +1$		Sum $= +1$	Sum $= +1$								Sum $= m$

add. * / add. * / add. * / add. * / add. *

Check-equations X.

			a_s	a_s
		b'_s	$b'_s a_b$	$b'_s s_b$
	c''_s	$c''_s b'_c$	$c''_s a'_c$	$c''_s s'_c$
$+1$	c''_d	b''_d	a''_d	s''_d
$+1$	$+1$	$+1$	$+1$	m

add. *

Solution.

$$\begin{aligned} [zz]\,X + [z\beta]\,Y + [z\gamma]\,Z + [z\delta]\,U &= x \\ [\beta\beta] + [\beta\gamma] + [\beta\delta] &= y \\ [\gamma\gamma] + [\gamma\delta] &= z \\ [\delta\delta] &= u \end{aligned}$$

weights.
1
$\frac{1}{[zz]}$
1
$\frac{1}{[\beta\beta]}$
1
$\frac{1}{[\gamma\gamma]}$
$\frac{1}{[\delta\delta]} = [dd3]$

Left hand members / *Right hand members*

	x	y	z	u	s	U	Z	Y	X	σ	
$E_4 = S_4$	1	$\frac{c''_d}{[dd3]} = [\delta\gamma]$	$\frac{b''_d}{[dd3]} = [\delta\beta]$	$\frac{a''_d}{[dd3]} = [\delta z]$	$\frac{s''_d}{[dd3]} = [\delta\sigma]$	*					$[ds]\ [\delta\sigma]$
E'_3		1	b'_c	a'_c	s'_c	*					
	c''_d	$\frac{[cc2]}{c''_d[\delta\gamma]}$	$\frac{[cc2]}{c''_d[\delta\beta]}$	$\frac{[cc2]}{c''_d[\delta z]}$	$\frac{[cc2]}{c''_d[\delta\sigma]}$						
S_3		$[\gamma\gamma]$	$[\gamma\beta]$	$[\gamma z]$	$[\gamma\sigma]$	add. *					$[cs]\ [\gamma\sigma]$
E_2			1	a_b	s_b						
	b'_d	b'_c	$\frac{[bb1]}{b'_d[\delta\beta]}$	$\frac{[bb1]}{b'_d[\delta z]}$	$\frac{[bb1]}{b'_d[\delta\sigma]}$						
			$b'_c[\gamma\beta]$	$b'_c[\gamma z]$	$b'_c[\gamma\sigma]$						
S_2			$[\beta\beta]$	$[\beta z]$	$[\beta\sigma]$	add. *					$[bs]\ [\beta\sigma]$
E_1				1	1						
	a_d	a_c		$\frac{[aa]}{a_d[\delta z]}$	$\frac{[aa]}{a_d[\delta\sigma]}$						
				$a_c[\gamma z]$	$a_c[\gamma\sigma]$						
			a_b	$a_b[\beta z]$	$a_b[\beta\sigma]$						
S_1				$[zz]$	$[z\sigma]$	add. *					$[as]\ [z\sigma]$
						Sum $= [s\sigma] = m$ *					

NUMERICAL EXAMPLE.

(The values printed in smaller type are those resulting from the check-equations.)

Normal Equations

$$\begin{aligned} +10\,x + 3\,y + 2\,z - 3\,u &= X \\ +12 \quad +4 \quad -2 &= Y \\ +8 \quad -1 &= Z \\ +7 &= U \end{aligned}$$

Left hand members / *Right hand members*

x	y	z	u	s		U	Z	Y	X	σ	
$+10.00$	$+3.00$	$+2.00$	-3.00	$+12.00$					$+1.000$	$+1.000$	$+1.200$
	$+12.00$	$+4.00$	-2.00	$+17.00$				$+1$		$+1.000$	
-0.300	$-.90$	$-.60$	$+.90$	-3.60					$+0.300$	$-.300$	
	$+11.10$	$+3.40$	-1.10	$+13.40$	$+13.40$			$+1.000$	-0.300	$+0.700$	$+0.700\ +0.845$
		$+8.00$	-1.00	$+13.00$			$+1$			$+1.000$	
-0.200		$-.40$	$+.60$	-2.40					-0.200	$-.200$	
	-0.306	-1.04	$+.34$	-4.10				-0.306	$+92$	$-.214$	
		$+6.56$	-0.06	$+6.50$	$+6.50$		$+1.000$	-0.306	-0.108	$+0.586$	$+0.586\ +0.581$
			$+7.00$	$+1.00$		$+1$				$+1.000$	
$+0.300$			$-.90$	$+3.60$					$+0.300$	$+.300$	
	$+0.099$		$-.11$	$+1.33$				$+0.099$	-30	$+69$	
		$+0.009$	-0	$+6$			$+0.009$	-3	-1	$+5$	
			$+5.99$	$+5.99$		$+1.000$	$+0.009$	$+0.096$	$+0.269$	$+1.374$	$+1.374\ +1.374$
											$+4.000$
$+1.200$	$+1.207$	$+0.991$									
$+1.000$	$+1.000$	$+1.000$									

Check-equations X.

			$+1.200$	$+1.200$
		$+1.207$	-0.362	$+0.845$
	$+0.991$	-0.303	-0.107	$+0.581$
$+1.000$	$+0.009$	$+0.096$	$+0.269$	$+1.374$
$+1.000$	$+1.000$	$+1.000$	$+1.000$	$+4.000$

Solution.

$$\begin{aligned} +0.1220\,X - 0.0177\,Y - 0.0161\,Z + 0.0449\,U &= x \\ +0.1059 \quad -0.0465 \quad +0.0160 &= y \\ +0.1524 \quad +0.0015 &= z \\ +0.1669 &= u \end{aligned}$$

weights.
8.20
9.44
6.56
5.99

$+0.1669$	$+0.0015$	$+0.0160$	$+0.0449$	$+0.2293$	$+0.2293\ +0.229$
	$+0.1524$	-0.0466	-0.0165	$+0.0893$	$+0.0893$
$+0.009$	$+0$	$+1$	$+4$	$+21$	
	$+0.1524$	-0.0465	-0.0161	$+0.0914$	$+0.0913\ +1.188$
		$+0.0901$	-0.0270	$+0.0631$	$+0.0631$
$+0.099$		$+16$	$+44$	$+227$	
	-0.306	$+142$	$+49$	-280	
		$+0.1059$	-0.0177	$+0.0578$	$+0.0577\ +0.983$
			$+0.1000$	$+0.1000$	
$+0.300$			$+135$	$+688$	
	-0.200		$+32$	-183	
		-0.300	$+53$	-173	
			$+0.1220$	$+0.1332$	$+0.1331\ +1.598$
					$[s\sigma] = +3.998$

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